Solucionario

Geometría

3.°

Unidad 1

SEGMENTOS

PRACTIQUEMOS

Nivel 1 (página 7) Unidad 1

Comunicación matemática

1.

2.

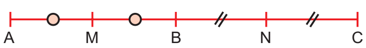
3.

Razonamiento y demostración

4. LP = 32
Piden: LN = 5 + 2k
5 + 2k + 7k = 32 LN = 5 + 2(3)
k = 3 LN = 11

Clave A

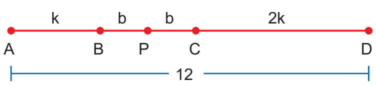
5.



AC = 12
2a + 2b = 12
a + b = 6; pero a + b = MN ⇒ MN = 6

Clave D

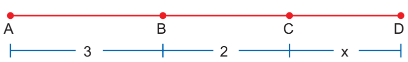
6.



AP = 12 ⇒ k + b = 12
piden BD = 2b + 2k
BD = 2(b + k) ⇒ BD = 2(12) ∴ BD = 24

Clave D

7.

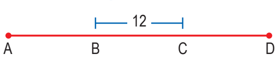


Del el dato: 4AB – AD = 4 + 2CD
Reemplazando: 4(3) – (3 + 2 + x) = 4 + 2(x)
12 – 5 – x = 4 + 2x
3x = 3 ⇒ x = 1
Piden AD:
AD = 3 + 2 + x ⇒ AD = 5 + 1 ⇒ AD = 6

Clave D

Resolución de problemas

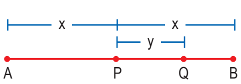
8.



Dato: AC = 3AB
⇒ AB + BC = 3AB ⇒ BC = 2AB
12 = 2AB ⇒ AB = 6
Luego: AC = AB + BC ⇒ AC = 6 + 12 = 18
Dato: 5AC = 3AD
5AC = 3(AC + CD)
2AC = 3CD ⇒ CD = $\frac{2}{3}$(18) = 12
∴ CD = 12

Clave D

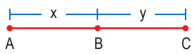
9.



$$\frac{AQ - QB}{PQ} = \frac{(x + y) - (x - y)}{y} = \frac{2y}{y} = 2$$

Clave D

10.



Dato: AB – BC = 6 ⇒ x – y = 6 ...(I)
Dato: AB + BC = 10 ⇒ x + y = 10 ...(II)
De (I) y (II): x = 8 ∧ y = 2
Nos piden: AB = x = 8 cm

Clave B

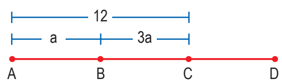
11.

Dato: $\frac{AB}{BC} = \frac{AD}{CD} \Rightarrow \frac{6}{2} = \frac{8 + x}{x} \Rightarrow x = 4$

Nos piden: AD = 8 + x = 8 + 4 ⇒ AD = 12

Clave B

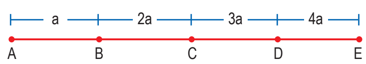
12.



Dato: AB = 1/3 BC
Sea: BC = 3a ⇒ AB = a
Del gráfico: AC = a + 3a = 12 ⇒ a = 3
Nos piden: AB = a = 3

Clave C

13.



$$AB = \frac{1}{2}BC = \frac{1}{3}CD = \frac{1}{4}DE$$

Sea: AB = a ⇒ BC = 2a; CD = 3a; DE = 4a
Dato: AC = 3a = 6 ⇒ a = 2
Nos piden: AE = 10a = 10(2) = 20

Clave C

Nivel 2 (página 7) Unidad 1

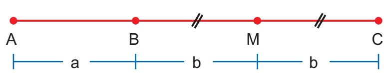
Comunicación matemática

14.

15.

Razonamiento y demostración

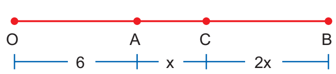
16.



Reemplazando: a + a + b + b = 12
2(a + b) = 12
a + b = 6
Pero a + b = AM ∴ AM = 6

Clave E

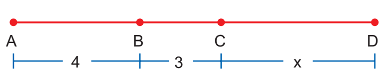
17.



OB = 15 Piden: OC
6 + x + 2x = 15 OC = 6 + x
3x = 9 OC = 6 + 3 ⇒ OC = 9
x = 3

Clave C

18.



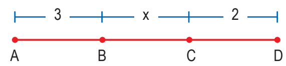
Reemplazando:

$\frac{3 + x}{5} = \frac{x}{3} \Rightarrow 9 + 3x = 5x$ ∴ x = 4,5

Clave D

Resolución de problemas

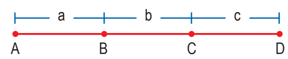
19.



Dato: 4BC + 5AD = 88
4x + 5(5 + x) = 88 ⇒ x = 7
Nos piden: AC = 3 + x = 3 + 7 = 10
∴ AC = 10

Clave D

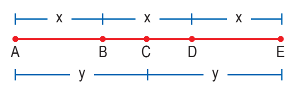
20.



Dato: 2AC = AB + AD – BC
Reemplazando:
2(a + b) = a + (a + b + c) – b
2a + 2b = 2a + c
c = 2b
Nos piden: $\frac{CD}{BC} = \frac{c}{b} = \frac{2b}{b} = 2$

Clave E

21.



Del gráfico se observa que:
3x = 2y ... (I)
Por dato:
AB + AC = 15
x + y = 15 ... (II)
De (I) y (II):
x = 6 ∧ y = 9
Nos piden: AE = 2y = 2 (9) = 18
∴ AE = 18

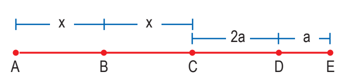
Clave A

22.

Dato: AC + BD = 32
(x + 8) + (8 + z) = 32
x + 16 + z = 32
x + z = 16
Piden: AD
AD = x + 8 + z = 16 + 8
∴ AD = 24

Clave C

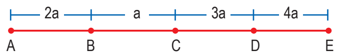
23.



Dato: 2DE = CD
Si: DE = a ⇒ CD = 2a ⇒ AB + AE = 6
x + 2x + 3a = 6
x + a = 2

Nos piden: AD = 2x + 2a = 2(x + a)
AD = 2(2) = 4

Clave A

24. 

$$\frac{1}{BC}=\frac{2}{AB}=\frac{3}{CD}=\frac{4}{DE}$$

$$BC=\frac{AB}{2}=\frac{CD}{3}=\frac{DE}{4}$$

Sea $BC = a \Rightarrow AB = 2a$; $CD = 3a$; $DE = 4a$

Además: $AE = 10a = 20 \Rightarrow a = 2$

Nos piden: $BC = a = 2$

Clave A

Nivel 3 (página 8) Unidad 1

Comunicación matemática

25.

26.

Razonamiento y demostración

27. Del gráfico: $2a = 2b + 9$

$a - b = 4{,}5$ ... (I)

Luego: $AE = AF + FE$

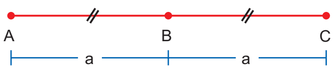
Piden: $FE \Rightarrow FE = AE - AF$

$FE = a - b$

De (I): $a - b = 4{,}5$

$\Rightarrow FE = 4{,}5$

Clave A

28. 

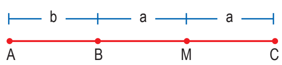
Reemplazando: $\frac{1}{a}-\frac{1}{2a}=\frac{1}{5}$

$5\left(\frac{2-1}{2a}\right)=1 \Rightarrow 2a = 5 \Rightarrow a = 2{,}5$

Piden $2a \Rightarrow 2a = 5$

Clave E

Resolución de problemas

29. 

Dato: $AB^2 + AC^2 = 8$

$b^2 + (b + 2a)^2 = 8$

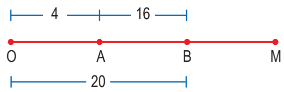
$b^2 + 2a^2 + 2ab = 4$

Nos piden:

$AM^2 + BM^2 = (b + a)^2 + a^2$

$AM^2 + BM^2 = b^2 + 2a^2 + 2ab = 4$

Clave D

30. 

Dato:

$3AB = 2(AM + BM)$

$3.16 = 2(AB + BM + BM)$

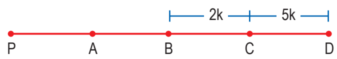
$24 = 16 + 2BM \Rightarrow BM = 4$

Nos piden:

$OM = OB + BM$

$OM = 20 + 4 = 24$ $\therefore OM = 24$

Clave B

31. 

Dato:

$7PC = 2PD + 5PB$

$7(PB + BC) = 2(PB + BD) + 5PB$

$7BC = 2BD$

$7BC = 2(BC + CD)$

$5BC = 2CD$

Sea $CD = 5k \wedge BC = 2k$

Dato:

$2AD + 5AB = 7$

$2(AB + BD) + 5AB = 7$

$2(AB + 7k) + 5AB = 7$

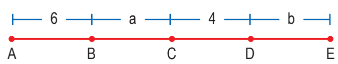
$AB + 2k = 1$

Nos piden:

$AC = AB + BC$

$AC = AB + 2k = 1$ $\therefore AC = 1$

Clave A

32. 

Dato: $(AB)(CD) = (BC)(AD)$

$6(4) = a(10 + a) \Rightarrow a = 2$

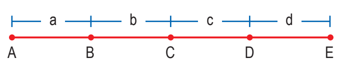
Dato: $DE = 2BC$

$b = 2a = 2(2) \Rightarrow b = 4$

Nos piden: $BE = a + b + 4 = 2 + 4 + 4$

$\therefore BE = 10$

Clave B

33. 

Dato:

$\frac{AE}{BD}=\frac{5}{3} \Rightarrow \frac{a+b+c+d}{b+c}=\frac{5}{3}$

$3(a + d) = 2(b + c)$ ... (I)

Dato:

$AC + BD + CE = 32$

$(a + b) + (b + c) + (c + d) = 32$

$a + 2(b + c) + d = 32$ ...(II)

Reemplazando (I) en (II):

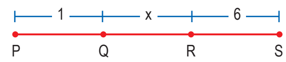
$4(a + d) = 32 \Rightarrow a + d = 8$

De (I): $3(8) = 2(b + c) \Rightarrow b + c = 12$

Nos piden: $BD = b + c = 12$

$\therefore BD = 12$

Clave A

34. 

$$\frac{1}{QR}+\frac{1}{PR}+\frac{1}{RS}=1$$

$$\frac{1}{x}+\frac{1}{x+1}+\frac{1}{6}=1$$

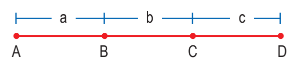
$$\frac{2x+1}{x(x+1)}=\frac{5}{6} \Rightarrow 12x + 6 = 5x^2 + 5x$$

$5x^2 - 7x = 6$

$x(5x - 7) = 3(2)$

$x = 2$ $\therefore x = 2$

Clave A

35. 

Por dato:

$\frac{AB}{BC}=\frac{AD}{CD} \Rightarrow \frac{a}{b}=\frac{a+b+c}{c}$ ...(I)

$(BC)(CD) = 63 \Rightarrow bc = 63$ ...(II)

$CD - BC = 18$

$c - b = 18$ ...(III)

Resolviendo (II) y (III):

$c = 21 \wedge b = 3$

Reemplazando en (I):

$\frac{a}{3}=\frac{a+24}{21} \Rightarrow a = 4$

Nos piden:

$AC = a + b = 4 + 3 = 7$

Clave C

PRACTIQUEMOS

Nivel 1 (página 11) Unidad 1

Comunicación matemática

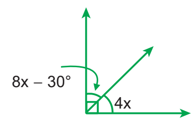
1.

2.

3.

Razonamiento y demostración

4.



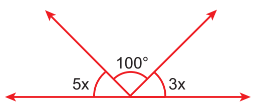
Del gráfico:

$8x - 30° + 4x = 90° \Rightarrow 12x = 120°$

$\therefore x = 10°$

Clave A

5.



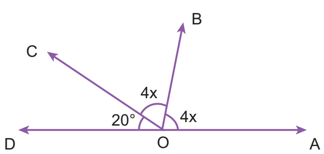
Del gráfico:

$5x + 100° + 3x = 180° \Rightarrow 8x = 80°$

$\therefore x = 10°$

Clave B

6.



Dato: $\overleftrightarrow{OB}$ es bisectriz del $\angle$ AOC:

$\Rightarrow m\angle AOB = m\angle BOC$

Luego:

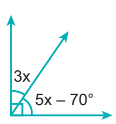
$20° + 4x + 4x = 180°$

$8x = 160°$

$\therefore x = 20°$

Clave C

7.



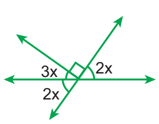
Del gráfico:

$3x + 5x - 70° = 90° \Rightarrow 8x = 160°$

$\therefore x = 20°$

Clave B

8.



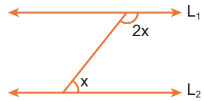
Del gráfico:

$3x + 2x + 90° = 180° \Rightarrow 5x = 90°$

$\therefore x = 18°$

Clave D

9.

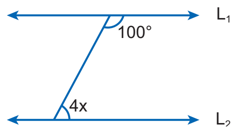


Del gráfico:

$2x + x = 180° \Rightarrow 3x = 180° \quad \therefore x = 60°$

Clave C

10.



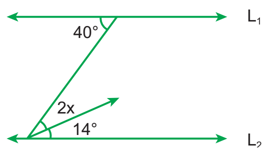
Del gráfico:

$4x + 100° = 180° \Rightarrow 4x = 80°$

$\therefore x = 20°$

Clave B

11.

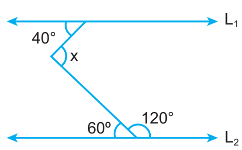


Del gráfico:

$40° = 2x + 14° \quad \therefore x = 13°$

Clave C

12.

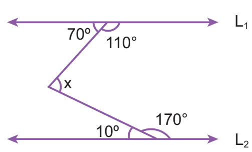


Del gráfico:

$x = 40° + 60° \quad \therefore x = 100°$

Clave B

13.

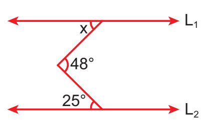


Del gráfico:

$x = 70° + 10° \quad \therefore x = 80°$

Clave C

14.

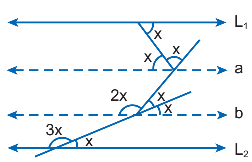


Del gráfico:

$x + 25° = 48° \quad \therefore x = 23°$

Clave C

15.



Trazamos $\overleftrightarrow{a}$ y $\overleftrightarrow{b}$ paralelas a $\overleftrightarrow{L_1}$ y $\overleftrightarrow{L_2}$.

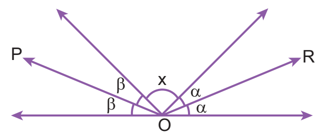
Luego: $4x = 180°$

$\therefore x = 45°$

Clave E

Resolución de problemas

16.



Dato: $m\angle POR = 100° \Rightarrow \alpha + \beta + x = 100°$

$2\alpha + 2\beta + 2x = 200°$

Se cumple: $2\alpha + 2\beta + x = 180°$

$\therefore x = 20°$

Clave B

17. Sea el ángulo: x

$x - 2(90° - x) = 30° \Rightarrow 3x - 180° = 30°$

$\Rightarrow 3x = 210° \Rightarrow x = 70°$

Clave A

18. Sea el ángulo: x

$x - (90° - x) = 10° \Rightarrow 2x - 90° = 10°$

$2x = 100° \Rightarrow \therefore x = 50°$

Clave E

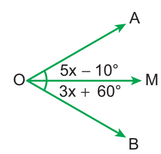
19. Sea el ángulo: x

$x = 3(180° - x) \Rightarrow x = 540° - 3x$

$4x = 540° \quad \therefore x = 135°$

Clave D

20.



Como $\overleftrightarrow{OM}$ es bisectriz del $\angle AOB$:

$m\angle AOM = m\angle MOB$

$5x - 10° = 3x + 60°$

$2x = 70° \quad \therefore x = 35°$

Clave C

21. $\frac{1}{2}\left(\frac{1}{3}\right)(90° - (180° - 102°)) = \frac{1}{6}(90° - 78°)$

$\Rightarrow \frac{1}{6}(12°) = 2°$

Clave B

22. Sea el ángulo: x

$(180° - x) - 4(90° - x) = 2(90° - x)$

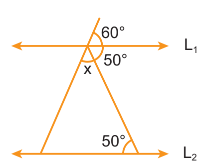
$180° - x - 360° + 4x = 180° - 2x$

$3x - 180° = 180° - 2x$

$5x = 360° \quad \therefore x = 72°$

Clave D

23.



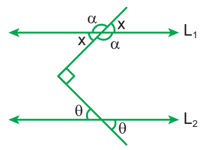
Del gráfico:

$x + 50° + 60° = 180°$

$x + 110° = 180° \quad \therefore x = 70°$

Clave B

24.



Dato: $\alpha + \theta = 142°$ ...(I)

Del gráfico: $x + \theta = 90°$

$180° - \alpha + \theta = 90°$

$90° = \alpha - \theta$ ...(II)

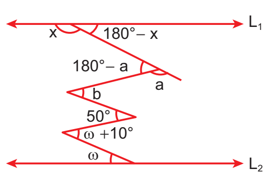
De (I) y (II): $232° = 2\alpha \Rightarrow 116° = \alpha$

Luego: $x = 180° - \alpha \Rightarrow x = 180° - 116°$

$\therefore x = 64°$

Clave A

25.



Dato: $a + b = 160°$

Del gráfico:

$180° - x + b + \omega + 10° = 180° - a + 50° + \omega$

$a + b - 40° = x$

$160° - 40° = x$

$\therefore x = 120°$

Clave E

Nivel 2 (página 12) Unidad 1

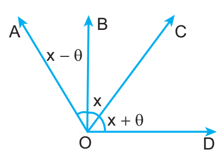
Comunicación matemática

26.

27.

Razonamiento y demostración

28.



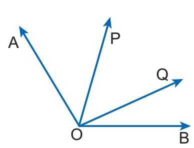
Dato: $m\angle AOD = 102°$

$x - \theta + x + x + \theta = 102°$

$3x = 102°$ $\therefore x = 34°$

Clave E

29.



Dato: $m\angle AOB = 100°$; $m\angle AOQ = 56°$ y $m\angle POQ = 74°$

$m\angle AOQ + m\angle POB = 130°$

$m\angle AOQ + m\angle POQ + m\angle QOB = 130°$

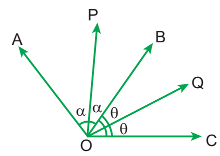
$m\angle AOB + m\angle POQ = 130°$

$100° + m\angle POQ = 130°$

$\therefore m\angle POQ = 30°$

Clave A

30.



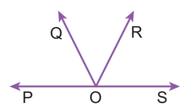
Dato: $m\angle AOC = 160°$

$2\alpha + 2\theta = 160°$

$\alpha + \theta = 80$ $\therefore m\angle POQ = 80°$

Clave D

31.



Dato: $m\angle POR + m\angle QOS = 240°$

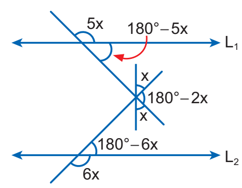
$m\angle POQ + m\angle QOR + m\angle QOS = 240°$

$m\angle QOR + 180° = 240°$

$\therefore m\angle QOR = 60°$

Clave B

32.



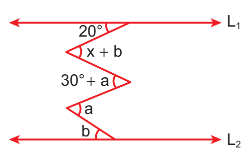
Del gráfico:

$180° - 5x + 180° - 6x = 180° - 2x$

$180° = 9x$ $\therefore x = 20°$

Clave C

33.

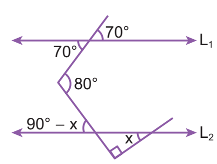


Por propiedad:

$x + b + a = 20° + 30° + a + b$ $\therefore x = 50°$

Clave B

34.



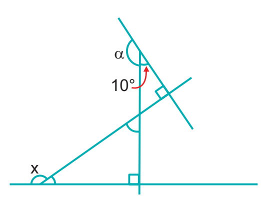
Del gráfico:

$70° + 90° - x = 80°$

$\therefore x = 80°$

Clave D

35.



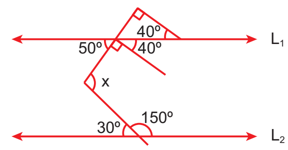
Del gráfico:

$\alpha + 10° = 180° \Rightarrow \alpha = 170°$

Por propiedad: $x = \alpha$ $\therefore x = 170°$

Clave D

36.

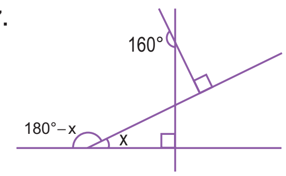


Del gráfico: $x = 50° + 30°$

$\therefore x = 80°$

Clave B

37.



Por propiedad:

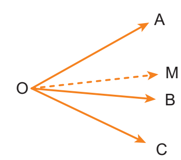
$180° - x = 160°$

$\therefore x = 20°$

Clave C

Resolución de problemas

38.



Dato: $m\angle AOB - m\angle BOC = 42°$ ...(1)

Como $\overleftrightarrow{OM}$ es bisectriz del $\angle AOC$:

$m\angle AOM = m\angle MOC$

Reemplaando en (1):

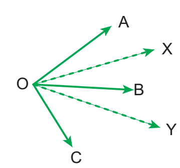
$m\angle AOM + m\angle MOB - (m\angle MOC - m\angle MOB) = 42°$

$2m\angle MOB + m\angle AOM - m\angle MOC = 42°$

$\therefore m\angle MOB = 21°$

Clave A

39.



Como $\overleftrightarrow{OX}$ y $\overleftrightarrow{OY}$ son bisectrices de los ángulos AOB y AOC, entonces:

$m\angle AOX = \dfrac{m\angle AOB}{2} \wedge m\angle AOY = \dfrac{m\angle AOC}{2}$

$m\angle AOY - m\angle AOX = \dfrac{m\angle AOC - m\angle AOB}{2}$

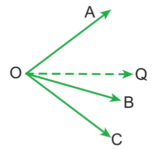
$m\angle XOY = \dfrac{m\angle BOC}{2}$

$32° = \dfrac{m\angle BOC}{2}$

$\therefore m\angle BOC = 64°$

Clave B

40.



Como $\overleftrightarrow{OQ}$ es la bisectriz del $\angle AOC$:

$\Rightarrow m\angle AOQ = m\angle QOC$

Dato: $m\angle AOB - m\angle BOC = 30°$

$m\angle AOQ + m\angle QOB - (m\angle QOC - m\angle QOB) = 30°$

$2m\angle QOB + m\angle AOQ - m\angle QOC = 30°$

$\therefore\ m\angle QOB = 15°$

Clave E

41. Dato:

$\dfrac{m\angle A}{4} = \dfrac{m\angle B}{6} = \dfrac{m\angle C}{5}$

$m\angle A = \dfrac{2}{3}m\angle B$ y $m\angle C = \dfrac{5}{6}m\angle B$

$\Rightarrow m\angle B > m\angle C > m\angle A$

Dato:

$90° - (m\angle A + m\angle B + m\angle C) = 15°$

$90° - \left(\dfrac{2}{3}m\angle B + m\angle B + \dfrac{5}{6}m\angle B\right) = 15°$

$90 - \dfrac{15}{6}\ m\angle B = 15°$

$\dfrac{15m\angle B}{6} = 75°$

$\therefore\ m\angle B = 30°$

Clave B

42. $(180° - x) - (2(90° - x) - 30°) = \dfrac{3}{11}(180° - x)$

$180° - x - (150° - 2x) = \dfrac{3}{11}(180° - x)$

$30° + x = \dfrac{3}{11}\ (180° - x)$

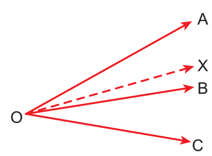
$330° + 11x = 540° - 3x$

$14x = 210°$

$\therefore x = 15°$

Clave A

43.



Como $\overleftrightarrow{OX}$ es bisectriz del $\angle AOC$

$\Rightarrow m\angle AOX = m\angle COX$

Dato: $m\angle AOB - m\angle BOC = 32°$

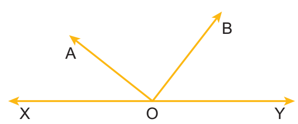
$m\angle AOX + m\angle BOX - (m\angle COX - m\angle BOX) = 32°$

$2m\angle BOX + m\angle AOX - m\angle COX = 32°$

$\therefore m\angle BOX = 16°$

Clave C

44.



Dato: $m\angle AOX = 60°$

$m\angle BOY = 180° - 3m\angle BOA$

Y se cumple:

$m\angle AOX + m\angle BOA + m\angle BOY = 180°$

$60° + m\angle AOB + 180° - 3m\angle AOB = 180°$

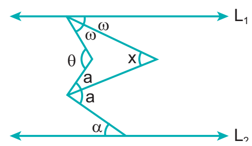
$60° - 2m\angle AOB = 0°$

$2m\angle AOB = 60°$

$\therefore m\angle AOB = 30°$

Clave B

45.



Dato: $\theta - \alpha = \dfrac{x}{2} + 45°$

Del gráfico:

$\omega + a = x + \alpha$ ...(I)

$2\omega + 2a = \theta + \alpha$ ...(II)

Luego:

$2x + 2\alpha = 2\omega + 2a$

$2x + 2\alpha = \theta + \alpha$

$2x = \theta - \alpha$

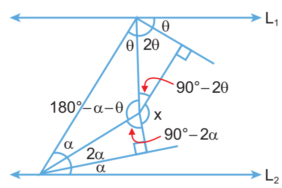
$2x = \dfrac{x}{2} + 45°$

$\dfrac{3x}{2} = 45°$

$\therefore x = 30°$

Clave C

46.



$4\alpha + 4\theta = 180°$

$\alpha + \theta = 45°$

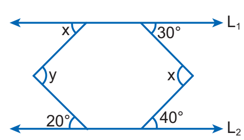
$x + 90° - 2\theta + 180° - \alpha - \theta + 90° - 2\alpha = 360°$

$x = 3\alpha + 3\theta$

$\therefore x = 135°$

Clave A

47.



$x = 30° + 40° = 70°$

$y = x + 20° = 70° + 20° = 90°$

$\therefore x + y = 160°$

Clave D

Nivel 3 (página 13) Unidad 1

Comunicación matemática

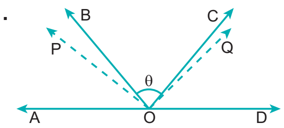
48.

49.

50.

Razonamiento y demostración

51.



Sea $\overleftrightarrow{OQ}$ es bisectriz del $\angle BOD$:

$\Rightarrow m\angle DOQ = \dfrac{m\angle DOB}{2}$ ...(I)

Sea $\overleftrightarrow{OP}$ es bisectriz del $\angle AOC$:

$\Rightarrow m\angle AOP = \dfrac{m\angle AOC}{2}$ ...(II)

Sumando (II) y (I):

$m\angle AOP + m\angle DOQ = \dfrac{m\angle AOD + m\angle BOC}{2}$

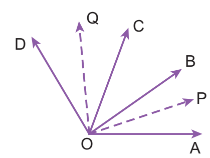
$m\angle AOD - m\angle POQ = \dfrac{m\angle AOD + m\angle BOC}{2}$

$180° - m\angle POQ = 90° + \dfrac{\theta}{2}$

$\therefore\ m\angle POQ = 90° - \dfrac{\theta}{2}$

Clave C

52.



Como $\overleftrightarrow{OP}$ y $\overleftrightarrow{OQ}$ son bisectrices de los $\angle BOA$ y $\angle DOC$, entonces:

$m\angle QOC = \dfrac{m\angle DOC}{2}$ y $m\angle BOP = \dfrac{m\angle BOA}{2}$

Dato: $m\angle POQ = 90°$

$m\angle QOC + m\angle COB + m\angle BOP = 90°$

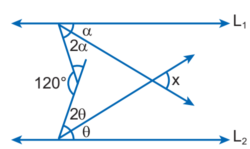
$2m\angle QOC + 2m\angle COB + 2m\angle BOP = 180°$

$m\angle DOC + m\angle COB + m\angle COB + m\angle BOA = 180°$

$\therefore m\angle DOB + m\angle COA = 180°$

Clave A

53.



Del gráfico:

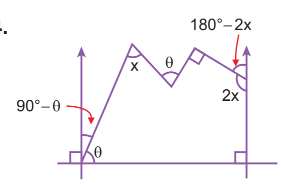
$3\alpha + 3\theta = 120°\quad \wedge \quad x = \alpha + \theta$

$\alpha + \theta = 40°$

$\therefore\ x = 40°$

Clave B

54.



Por propiedad:

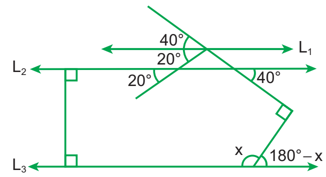
$90° - \theta + \theta + 180° - 2x = x + 90°$

$180° = 3x$

$60° = x$

Clave E

55.



Trazamos $\overleftrightarrow{L_1} \parallel \overleftrightarrow{L_2}$

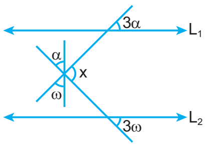
Luego:

$40° + 180° - x = 90°$

$\therefore x = 130°$

Clave C

56.



Del gráfico:

$x = 3\alpha + 3\omega \Rightarrow \frac{x}{3} = \alpha + \omega \Rightarrow \frac{x}{3} = \alpha + \omega$

Además:

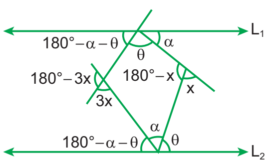
$x + \alpha + \omega = 180°$

$x + \frac{x}{3} = 180°$

$\therefore x = 135°$

Clave C

57.



Del gráfico:

$\alpha + \theta = 180° - x$; y por propiedad:

$180° - \alpha - \theta + 180° - \alpha - \theta = 180° - 3x$

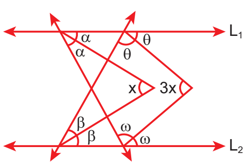
$180° - 2(180° - x) + 3x = 0$

$5x = 180°$

$\therefore x = 36°$

Clave E

58.



Del gráfico:

$2\alpha + 2\omega = 180° \quad \wedge \quad 2\theta + 2\beta = 180°$

$\alpha + \omega = 90° \qquad \theta + \beta = 90°$

Luego:

$x = \alpha + \beta$

$3x = \theta + \omega$

Entonces:

$4x = \alpha + \beta + \theta + \omega$

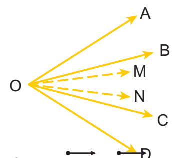
$4x = 90° + 90° = 180°$

$\therefore x = 45°$

Clave C

Resolución de problemas

59.



Como $\overleftrightarrow{OM}$ y $\overleftrightarrow{ON}$ son bisectrices de los $\angle AOC$ y $\angle BOD$, entonces:

$m\angle AOM = m\angle MOC \wedge m\angle BON = m\angle NOD$

Dato: $m\angle AOB + m\angle COD = 152°$

$m\angle AOM - m\angle BOM + m\angle NOD - m\angle NOC = 152°$

$m\angle MOC - m\angle NOC + m\angle BON - m\angle BOM = 152°$

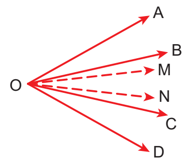
$m\angle MON + m\angle MON = 152°$

$2m\angle MON = 152°$

$\therefore m\angle MON = 76°$

Clave A

60.



Como $\overleftrightarrow{OM}$ y $\overleftrightarrow{ON}$ son bisectrices de los $\angle AOC$ y $\angle BOD$:

$m\angle AOM = m\angle MOC \wedge m\angle BON = m\angle NOD$

Dato: $m\angle AOB = 98°$

$m\angle COD = 98°$

$m\angle AOB + m\angle COD = 196°$

$m\angle AOM - m\angle BOM + m\angle NOD - m\angle NOC = 196°$

$m\angle MOC - m\angle NOC + m\angle BON - m\angle BOM = 196°$

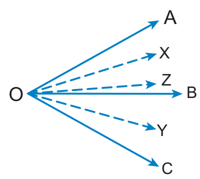
$m\angle MON + m\angle MON = 196°$

$2m\angle MON = 196°$

$\therefore m\angle MON = 98°$

Clave E

61.



Como $\overleftrightarrow{OX}$, $\overleftrightarrow{OY}$ y $\overleftrightarrow{OZ}$, son bisectrices de los $\angle AOB$, $\angle BOC$, $\angle XOY$, entonces:

$m\angle AOX = m\angle BOX = \frac{m\angle AOB}{2}$

$m\angle BOY = m\angle COY = \frac{m\angle BOC}{2}$

$m\angle XOZ = m\angle YOZ = \frac{m\angle XOY}{2}$

Dato: $m\angle AOB - m\angle BOC = 26°$

$2m\angle BOX - 2m\angle BOY = 26°$

$m\angle BOX + m\angle BOY = 13°$

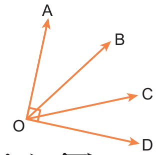
$m\angle XOZ + m\angle BOZ - (m\angle YOZ - m\angle BOZ) = 13°$

$2m\angle BOZ + m\angle XOZ - m\angle YOZ = 13°$

$\therefore m\angle BOZ = 6°30'$

Clave C

62.



Dato: $\overleftrightarrow{OA} \perp \overleftrightarrow{OD}$:

$m\angle AOD = 90°$

$m\angle AOC + m\angle BOD = 140°$

$m\angle AOC + m\angle COD + m\angle BOC = 140°$

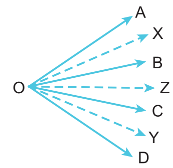
$m\angle AOD + m\angle BOC = 140°$

$90° + m\angle BOC = 140°$

$\therefore m\angle BOC = 50°$

Clave A

63.



Como $\overleftrightarrow{OX}$, $\overleftrightarrow{OY}$ y $\overleftrightarrow{OZ}$ son bisectrices de los ángulos AOB, COD y XOY, entonces:

$m\angle AOX = m\angle BOX$

$m\angle COY = m\angle DOY$

$m\angle XOZ = m\angle YOZ$

Dato: $m\angle BOY - m\angle AOX = 36°$

$m\angle BOZ + m\angle YOZ - (m\angle BOX) = 36°$

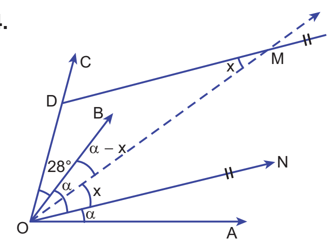
$m\angle BOZ + m\angle YOZ - (m\angle XOZ - m\angle BOZ) = 36°$

$2m\angle BOZ + m\angle YOZ - m\angle XOZ = 36°$

$\therefore m\angle BOZ = 18°$

Clave C

64.



$\overleftrightarrow{ON}$ y $\overleftrightarrow{OM}$ son bisectrices de los ángulos AOB y AOC, respectivamente, además $\overline{DM} \parallel \overleftrightarrow{ON}$.

Luego:

$m\angle COM = m\angle MOA$

$28° + \alpha - x = x + \alpha$

$28° = 2x$

$\therefore x = 14°$

Clave B

PRACTIQUEMOS

Nivel 1 (página 17) Unidad 1

Comunicación matemática

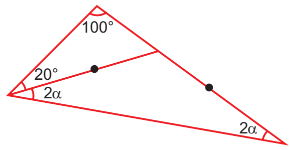
1.

2. **Clave E**

3.

Razonamiento y demostración

4. Piden: α



Del gráfico:

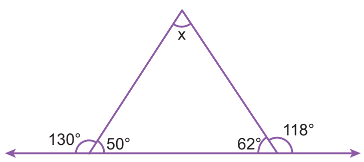
$20° + 2\alpha + 100° + 2\alpha = 180°$

$4\alpha = 60°$

$\therefore\ \alpha = 15°$

Clave B

5. Piden: x

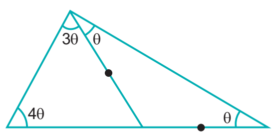


Del gráfico:

$50° + x + 62° = 180°$ $\quad\therefore\ x = 68°$

Clave B

6. Piden: θ



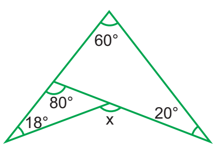
Del gráfico:

$4\theta + 4\theta + \theta = 180°$

$9\theta = 180°$ $\quad\therefore\ \theta = 20°$

Clave D

7. Piden: x

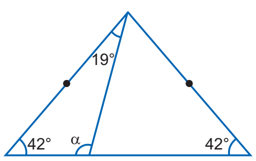


Del gráfico:

$18° + 80° = x$ $\quad\therefore\ x = 98°$

Clave C

8. Piden: α

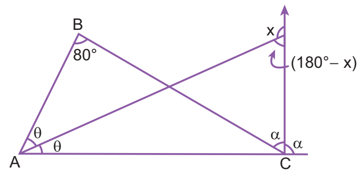


Del gráfico:

$42° + \alpha + 19° = 180°$ $\quad\therefore\ \alpha = 119°$

Clave A

9. Piden: x

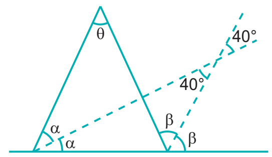


Por propiedad:

$180° - x = \dfrac{80°}{2}$ $\quad\therefore\ x = 140°$

Clave A

10. Piden: θ

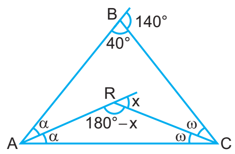


Por propiedad:

$40° = \dfrac{\theta}{2} \Rightarrow \theta = 80°$

Clave C

11. Piden: x

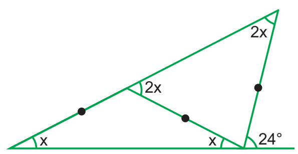


Por propiedad:

$180° - x = 90° + \dfrac{40°}{2} \Rightarrow x = 70°$

Clave D

12. Piden: x



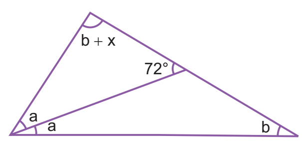
Del gráfico:

$x + 2x = 24°$

$3x = 24°$ $\quad\therefore\ x = 8°$

Clave A

13. Piden: x



Del gráfico:

$a + b = 72°$ $\quad$ ...(1)

También:

$a + b + x + 72° = 180°$

$a + b + x = 108°$ $\quad$ ...(2)

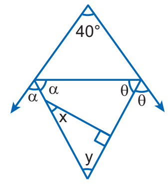
Reemplazando (1) en (2):

$72° + x = 108°$

$\therefore\ x = 36°$

Clave B

14. Piden: x



Por propiedad:

$y = 90° - \dfrac{40°}{2}$

$y = 70°$

Del gráfico:

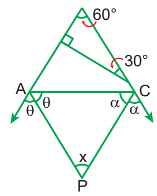
$x + y = 90°$

$x + 70° = 90°$

$\therefore\ x = 20°$

Clave C

15. Piden: x



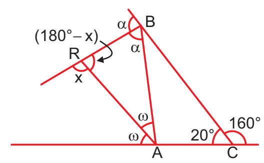
Por propiedad:

$x = 90° - \dfrac{60°}{2}$

$\Rightarrow\ x = 60°$

Clave E

16. Piden: x



Por propiedad:

$180° - x = 90° - \dfrac{20°}{2}$

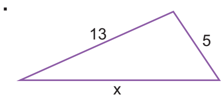
$180° - x = 80°$

$\therefore\ x = 100°$

Clave B

Resolución de problemas

17.



Por existencia de un triángulo:

$13 - 5 < x < 13 + 5$

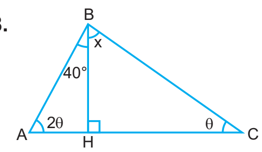
$8 < x < 18$

$x = \{9; 10; 11; 12; ...; 17\}$

$\therefore\ \Sigma$ valores enteros de x es: 117

Clave D

18.



Del gráfico: $2\theta + 40° = 90°$

$2\theta = 50°$

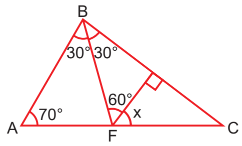
$\theta = 25°$

Luego: $x = 90° - \theta$

$x = 90° - 25°$ $\quad\therefore\ x = 65°$

Clave B

19.

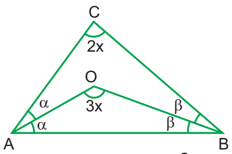


Por ángulo exterior: $60° + x = 70° + 30°$

$\therefore\ x = 40°$

Clave C

20.



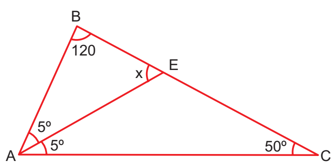
Por propiedad: $3x = 90° + \dfrac{2x}{2}$

$2x = 90°$

$\therefore\ x = 45°$

Clave E

21.

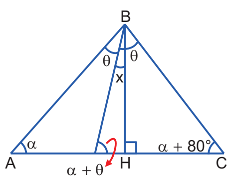


Por ángulo exterior: $x = 50° + 5°$

$\therefore\ x = 55°$

Clave D

22. Piden: x



Del gráfico:

$x + \alpha + \theta = 90°$ ...(1)

También:

$\alpha + 80° + 2\theta + \alpha = 180°$

$\alpha + \theta = 50°$ ...(2)

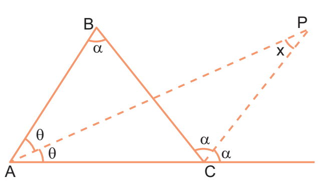
Reemplazando (2) en (1):

$x + 50° = 90°$

$\therefore\ x = 40°$

Clave B

23. Piden: $m\angle APC = x$



Por propiedad: $x = \dfrac{\alpha}{2}$ ...(1)

Por dato: $\alpha + x = 90°$

$\alpha = 90° - x$ ...(2)

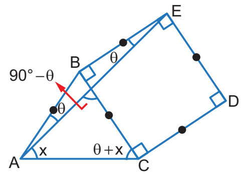
Reemplazando (2) en (1):

$x = \dfrac{90° - x}{2} \Rightarrow 2x = 90° - x \Rightarrow 3x = 90°$

$\therefore\ x = 30°$

Clave A

24. Piden: x



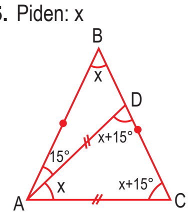
Del gráfico:

$90° - \theta + x + \theta + x = 180°$

$2x = 90°$ $\therefore\ x = 45°$

Clave E

25. Piden: x



Del gráfico:

$2(x + 15°) + x = 180°$

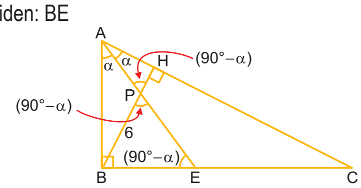
$2x + 30° + x = 180°$

$3x = 150°$

$\therefore\ x = 50°$

Clave E

26. Piden: BE



Del gráfico, el $\triangle BPE$ es isósceles.

$\therefore\ BE = 6$

Clave A

Nivel 2 (página 18) Unidad 1

Comunicación matemática

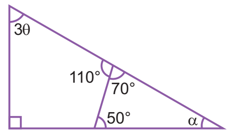
27.

28.

29.

Razonamiento y demostración

30.



$\alpha = 180° - 120° \Rightarrow \alpha = 60°$

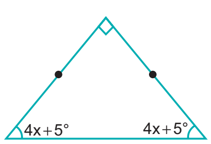
Piden θ: $3\theta + 60° = 90°$

$3\theta = 30°$

$\theta = 10°$

Clave E

31.



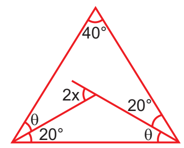
$8x + 10° = 90°$

$8x = 80°$

$\Rightarrow x = 10°$

Clave D

32.



Observación:

$20° + \theta = 2x$

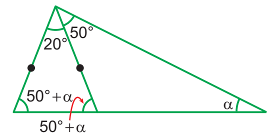
$40° + 2(\theta + 20°) = 180°$

$2(2x) = 140°$

$x = 35°$

Clave D

33.



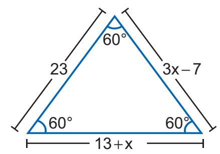
$100° + 2\alpha + 20° = 180°$

$2\alpha = 60°$

$\alpha = 30°$

Clave C

34.



$13 + x = 3x - 7$

$20 = 2x \Rightarrow 10 = x$

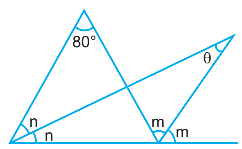
Perímetro (2p):

$2p = 23 + 23 + 23$

$2p = 69$

Clave E

35.



$\theta = \dfrac{80°}{2}$

$\theta = 40°$

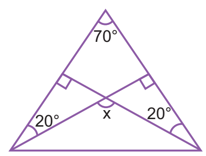
Piden: $C_{(\theta)}$

$C_{(\theta)} = 90° - 40°$

$C_{(\theta)} = 50°$

Clave C

36.

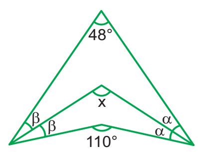


$x = 70° + 40°$

$x = 110°$

Clave D

37.

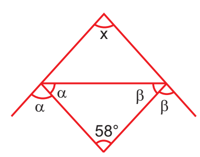


$x = \dfrac{110° + 48°}{2}$

$x = 79°$

Clave E

38.



$58° = 90° - \dfrac{x}{2}$

$\dfrac{x}{2} = 32°$

$x = 64°$

Piden $C_{(x)}$:

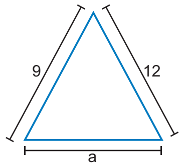
$C_{(x)} = 90° - 64°$

$C_{(x)} = 26°$

Clave E

Resolución de problemas

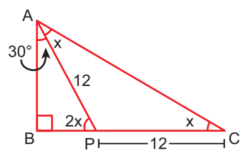
39. Piden: menor y mayor valor entero
Datos: los lados miden 9 y 12
Sea:



Por desigualdad triangular:
$12 - 9 < a < 9 + 12$
$3 < a < 21$
Por lo tanto:
Mayor valor entero: 20
Menor valor entero: 4

Clave B

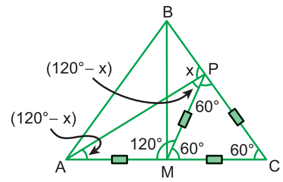
40. Piden: x



Del gráfico:
$30° + x + x = 90°$
$2x = 60°$
$\therefore\ x = 30°$

Clave A

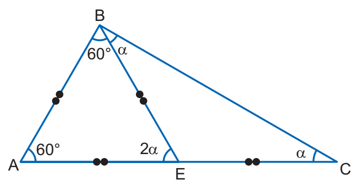
41.



Del gráfico: En $\triangle$APM:
$2(120° - x) + 120° = 180°$
$240° - 2x = 60°$
$180° = 2x$
$\therefore\ x = 90°$

Clave A

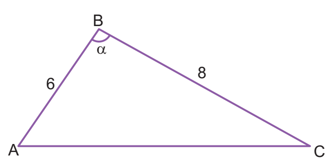
42.



Por dato: AB = AE = EC
Además: m∠BAC = 60°
Entonces, el $\triangle$ABE resulta equilátero.
$\Rightarrow$ BE = AE = AB
Del gráfico: $2\alpha = 60°$ $\therefore\ \alpha = 30°$

Clave C

43.

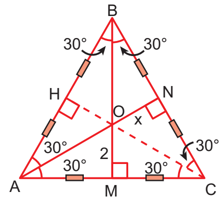


Por dato: $\alpha > 90°$
Si $\alpha = 90°$, entonces por el teorema de Pitágoras:
$(AC)^2 = 6^2 + 8^2 = 100 \Rightarrow AC = 10$
Pero $\alpha > 90°$, entonces: $AC > 10$ ...(1)
Por desigualdad triangular:
$8 - 6 < AC < 8 + 6 \Rightarrow AC < 14$ ...(2)

De (1) y (2): $10 < AC < 14$
Valores enteros de AC: {11; 12; 13}
Piden: $11 + 12 + 13 = 36$

Clave D

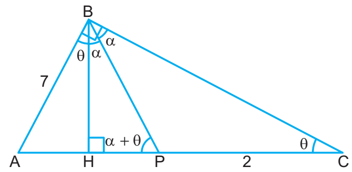
44.



Por dato: el $\triangle$ABC es equilátero
Entonces, O es el circuncentro, baricentro, ortocentro e incentro del $\triangle$ABC.
Por el teorema de la bisectriz: ON = OM
$\therefore$ x = 2

Clave B

45.



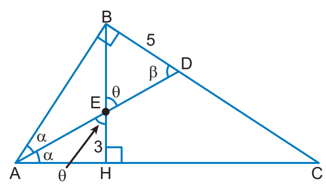
En el ⊿BHC: $2\alpha + \theta = 90°$
Entonces la m∠ABH = θ

Del gráfico, el $\triangle$ABP resulta isósceles.
$\Rightarrow$ AB = AP = 7

Piden:
AC = AP + PC = 7 + 2 = 9 $\therefore$ AC = 9

Clave C

46.



En el ⊿AHE: $\alpha + \theta = 90°$
$\theta = 90° - \alpha$ ...(1)

En el ⊿ABD: $\alpha + \beta = 90°$
$\beta = 90° - \alpha$ ...(2)

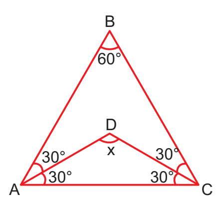
De (1) y (2): $\theta = \beta$
Entonces el $\triangle$EBD resulta isósceles.
$\Rightarrow$ BE = BD = 5
Piden:
BH = BE + EH = 5 + 3 $\therefore$ BH = 8

Clave B

Nivel 3 (página 19) Unidad 1

Comunicación matemática

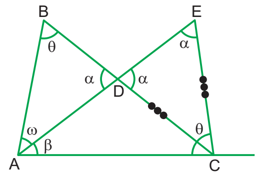
47.



Por dato: el $\triangle$ABC es equilátero
En el triángulo ADC:
$30° + x + 30° = 180°$
$x + 60° = 180°$ $\therefore\ x = 120°$

Clave D

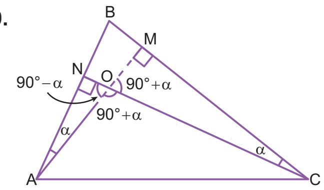
48.



Por dato: m∠ABC = m∠ACE
Además: DC = CE
En el $\triangle$ABD:
$\theta + \alpha + \omega = 180°$
$\Rightarrow \omega = 180° - (\theta + \alpha)$ ...(1)
En el $\triangle$AEC:
$\theta + \alpha + \beta = 180°$
$\Rightarrow \beta = 180° - (\theta + \alpha)$ ...(2)
De (1) y (2): $\omega = \beta$
Entonces $\overline{AD}$ es una bisectriz interior para el $\triangle$ABC.

Clave A

49.

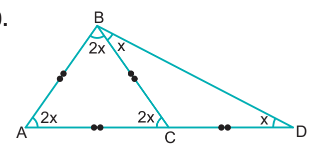


Del gráfico: prolongamos $\overline{AO}$ y $\overline{CO}$
Entonces, $\overline{AM}$ y $\overline{CN}$ serían alturas para el $\triangle$ABC.
Por lo tanto, O sería el ortocentro para el $\triangle$ABC.

Clave B

Razonamiento y demostración

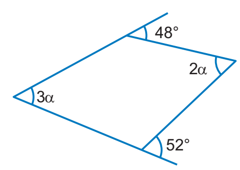
50.



Por dato: AB = BC = CD
Luego, el triángulo ABC resulta equilátero.
$\Rightarrow$ $2x = 60°$ $\therefore\ x = 30°$

Clave A

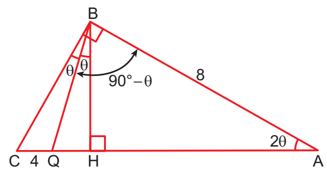
51.



Por propiedad, se cumple:
$3\alpha + 2\alpha = 48° + 52°$
$5\alpha = 100°$ $\therefore\ \alpha = 20°$

Clave B

52.



En el ◺BHQ: $m\angle BQH = 90° - \theta$

Entonces el △BAQ resulta isósceles.

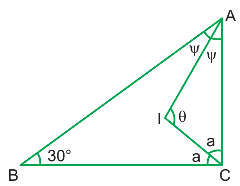
$\Rightarrow AB = AQ = 8$

Piden: $AC = AQ + QC = 8 + 4$

$\therefore AC = 12$

Clave B

53.



Por propiedad:

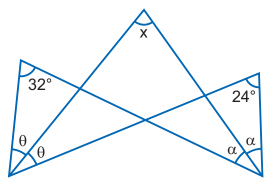
$\theta = 90° + \dfrac{30°}{2}$

$\theta = 90° + 15°$

$\therefore \theta = 105°$

Clave B

54.

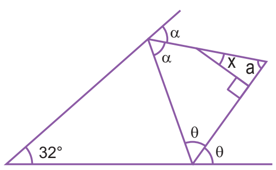


Por propiedad: $x = \dfrac{32° + 24°}{2} = \dfrac{56°}{2} = 28°$

$\therefore x = 28°$

Clave B

55.



Por propiedad:

$a = 90° - \dfrac{32°}{2} = 90° - 16° \Rightarrow a = 74°$

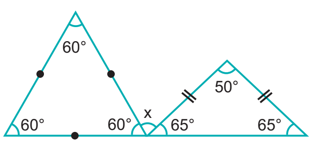
Del gráfico: $x + a = 90°$

$x + 74° = 90°$

$\therefore x = 16°$

Clave D

56.



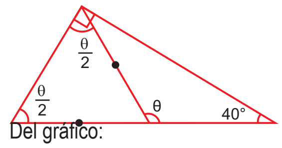
Del gráfico: $60° + x + 65° = 180°$

$x + 125° = 180°$

$\therefore x = 55°$

Clave A

57.



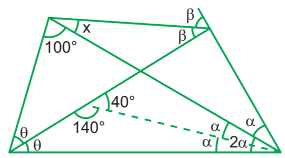
Del gráfico:

$\dfrac{\theta}{2} + 40° = 90° \Rightarrow \theta + 80° = 180°$

$\therefore \theta = 100°$

Clave A

58.

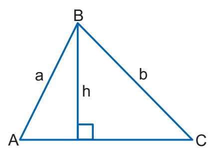


Por propiedad: $x = \dfrac{40°}{2}$ $\therefore x = 20°$

Clave D

Resolución de problemas

59.



Dato: $a + b = 28$ ... (1)

De la figura: $h < a \wedge h < b$

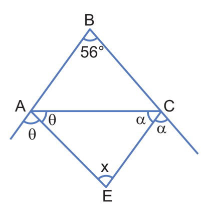
$\Rightarrow h < \dfrac{a+b}{2}$ ... (2)

(1) en (2): $h < 14$

$\therefore h = 13$ (mayor valor entero)

Clave B

60.



Por dato:

$m\angle A + 2m\angle B + m\angle C = 236°$ (–)

$m\angle A + m\angle B + m\angle C = 180°$

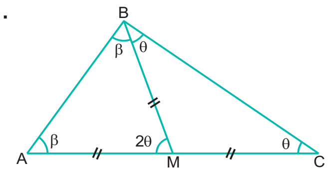
$\Rightarrow m\angle B = 56°$

Por propiedad de bisectrices:

$x = 90° - \dfrac{56°}{2} \Rightarrow x = 62°$

Clave A

61.



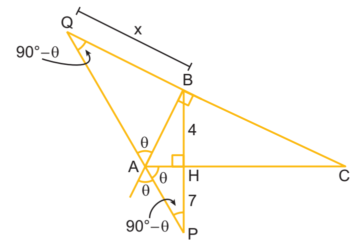
En el △ABM:

$2\theta + 2\beta = 180° \Rightarrow \theta + \beta = 90°$

$\therefore m\angle ABC = 90°$

Clave B

62.



De la figura:

$m\angle AQB = 90° - \theta$

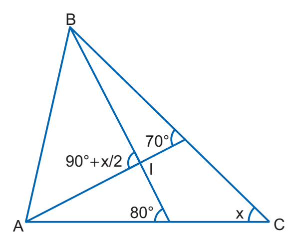
$m\angle APB = 90° - \theta$

$\Rightarrow$ △QBP es isósceles ($BQ = BP$)

$\therefore x = 11$

Clave D

63.



Nos piden: x

Como I es incentro: $m\angle AIB = 90° + \dfrac{x}{2}$

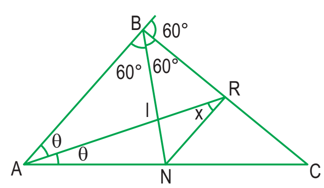
Luego:

$90° + \dfrac{x}{2} + x = 70° + 80°$

$\dfrac{3x}{2} = 60°$ $\therefore x = 40°$

Clave D

64.



R es excentro del △ABN.

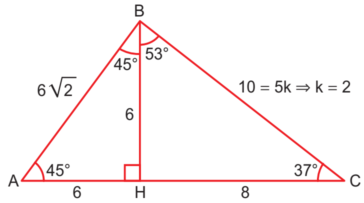
Por propiedad sabemos:

$x = \dfrac{60°}{2} = 30°$

Clave C

APLICAMOS LO APRENDIDO (página 21) Unidad 1

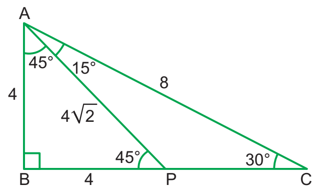
1.



Del gráfico:
AC = 6 + 8 = 14 $\therefore$ AC = 14

Clave E

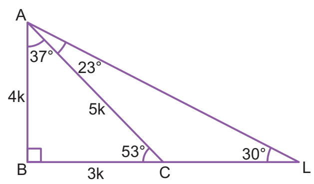
2.



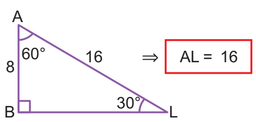
En el ◺ABC: AC = 2AB = 2(4)
$\therefore$ AC = 8

Clave B

3.

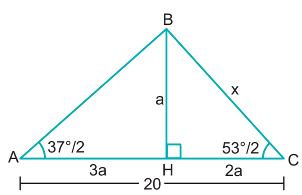


Por dato:
$4k + 3k + 5k = 24 \Rightarrow 12k = 24$
$k = 2$
Luego:



Clave A

4.

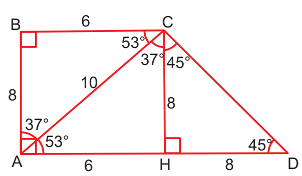


Del gráfico: $3a + 2a = 20$
$5a = 20 \Rightarrow a = 4$

Por el teorema de Pitágoras:
$x^2 = a^2 + 4a^2 \Rightarrow x^2 = 5a^2$
$x^2 = 5(4)^2 \quad \therefore x = 4\sqrt{5}$

Clave A

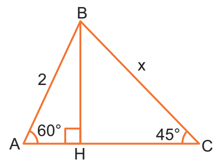
5.



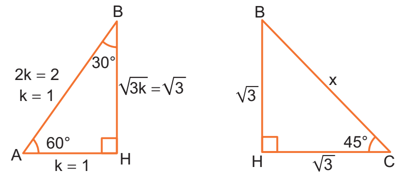
AD = 6 + 8 = 14

Clave B

6.



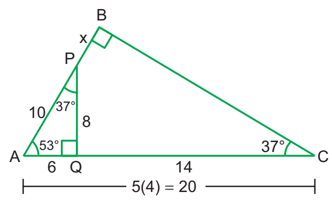
Analizando los triángulos rectángulos:



Luego: $x = \sqrt{3}(\sqrt{2})$
$\therefore x = \sqrt{6}$

Clave B

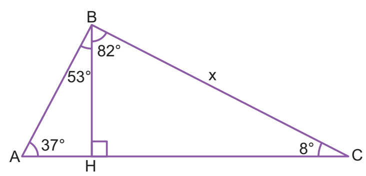
7.



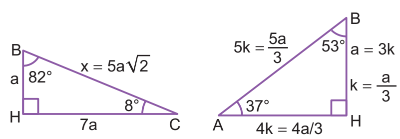
$10 + x = 3(4) \Rightarrow x = 12 - 10$
$\therefore x = 2$

Clave B

8.



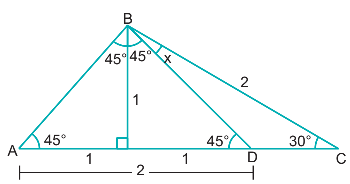
Analizamos los triángulos rectángulos:



Dato: AC + AB = 12
$\left(\frac{4a}{3} + 7a\right) + \frac{5a}{3} = 12$
$3a + 7a = 12$
$10a = 12 \Rightarrow a = 1{,}2$
$x = 5a\sqrt{2} = 6\sqrt{2}$
$\therefore x = 6\sqrt{2}$

Clave D

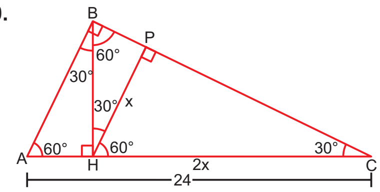
9.



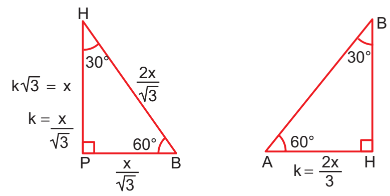
Por ángulo exterior:
$x + 30° = 45° \Rightarrow x = 15°$

Clave E

10.



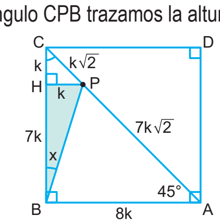
En el triángulo HPB: En el triángulo AHB:



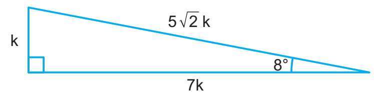
$\frac{2}{3}\sqrt{3}\,x = k\sqrt{3}$
$\frac{2x}{3} = k$
Luego:
$AC = \frac{2}{3}x + 2x = 24 \Rightarrow \frac{8x}{3} = 24$
$x = \frac{3(24)}{8} = 9 \quad \therefore x = 9$

Clave B

11. En el triángulo CPB trazamos la altura PH:



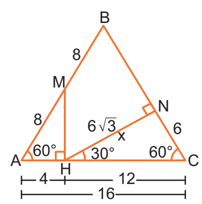
El ◺CHP es notable de 45°
$\therefore$ Si $CP = k\sqrt{2} \Rightarrow CH = HP = k$
Dato: $AP = 7PC \Rightarrow AP = 7k\sqrt{2}$
La diagonal $CA = 7k\sqrt{2} + k\sqrt{2}$
$CA = 8k\sqrt{2}$
El ◺CBA es notable de 45°:
$\therefore$ Si $CA = 8k\sqrt{2} \Rightarrow CB = 8k$
Pero $CH = k \Rightarrow HB = CB - CH \Rightarrow HB = 8k - k = 7k$
Luego el ◺PHB tiene lados k y 7k, entonces el triángulo es notable de 8° y 72°:



$\therefore x = 8°$

Clave E

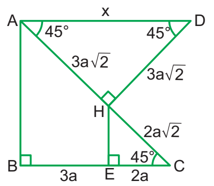
12. Como el △ABC es equilátero, entonces el ◺AHM y el ◺HNC son triángulos notables de 30° y 60°.
M es punto medio de $\overline{AB}$:
$\Rightarrow AM = MB = 8, \Rightarrow AH = 4$
Luego $AC = 16, \therefore HC = 16 - 4 = 12$



En el triángulo HNC:
Si HC = 12
$\Rightarrow HN = \frac{HC}{2}\sqrt{3}$
$HN = 6\sqrt{3}$
$x = 6\sqrt{3}$

Clave B

13. Los triángulos AHD y HEC son notables de 45°:



Si: $EC = 2a \Rightarrow HC = 2a\sqrt{2}$

$BC = 5a \Rightarrow AC = 5a\sqrt{2}$

Luego $AH = AC - HC = 3a\sqrt{2}$

En el ◺AHD x es hipotenusa:

$\Rightarrow x = \sqrt{2}(AH)$

$x = \sqrt{2}(3a\sqrt{2}) \Rightarrow x = 6a$

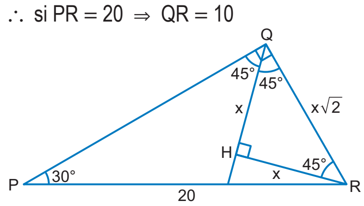
Pero $2a\sqrt{2} = 6 \Rightarrow a = \dfrac{3}{2}\sqrt{2}$

Reemplazamos: $x = 6\left(\dfrac{3}{2}\sqrt{2}\right) \Rightarrow x = 9\sqrt{2}$

Clave D

14. El triángulo PQR es notable de 30 y 60°,

$\therefore$ si $PR = 20 \Rightarrow QR = 10$



El ◺QHR es notable de 45°:

si $QH = x \Rightarrow QR = x\sqrt{2}$, pero $QR = 10$

$\Rightarrow x\sqrt{2} = 10 \Rightarrow x = 5\sqrt{2}$

Clave C

PRACTIQUEMOS

Nivel 1 (página 23) Unidad 1

Comunicación matemática

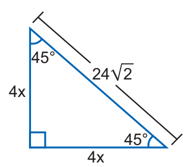
1.

2.

3.

Razonamiento y demostración

4.



Por triángulo notable:

$k\sqrt{2} = 24\sqrt{2}$

$k = 24$

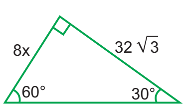
Piden x:

$4x = k$

$4x = 24 \Rightarrow x = 6$

Clave E

5.



Por triángulo notable:

$k\sqrt{3} = 32\sqrt{3}$

$k = 32$

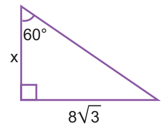
Piden x:

$8x = k$

$8x = 32 \Rightarrow x = 4$

Clave A

6.



Por triángulo notable:

$k\sqrt{3} = 8\sqrt{3}$

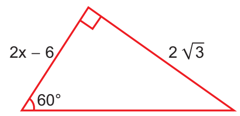
$k = 8$

Piden x:

$x = k \Rightarrow x = 8$

Clave E

7.



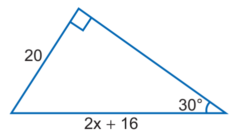
Por triángulo notable: $k\sqrt{3} = 2\sqrt{3}$

$k = 2$

Piden x: $2x - 6 = k \Rightarrow 2x = 8 \Rightarrow x = 4$

Clave E

8.



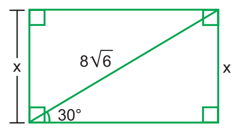
Por triángulo notable: $k = 20$

Piden x: $2x + 16 = 2k$

$2x + 16 = 40 \Rightarrow x = 12$

Clave E

9.

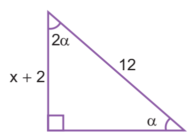


Por triángulo notable:

$8\sqrt{6} = 2x \Rightarrow x = 4\sqrt{6}$

Clave E

10.



$3\alpha = 90°$

$\alpha = 30°$

Por triángulo notable:

$2k = 12$

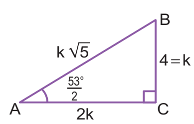
$k = 6$

Piden x:

$\Rightarrow x + 2 = k \Rightarrow x = 4$

Clave E

11.



Del gráfico:

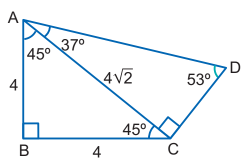
$BC = k = 4$

$AC = 2k = 2(4) = 8$

$\therefore AC = 8$ m

Clave B

12.



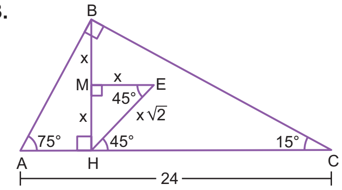
$AC = 4(\sqrt{2}) = 4\sqrt{2}$

$AD = \left(\dfrac{4\sqrt{2}}{4}\right)5 \quad \therefore AD = 5\sqrt{2}$ m

Clave A

Resolución de problemas

13.



Por dato: M es punto medio de $\overline{BH}$.

Sea: $MH = x$

Por propiedad: $BH = \dfrac{AC}{4} \Rightarrow 2x = \dfrac{24}{4}$

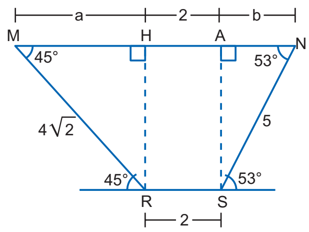
$2x = 6 \Rightarrow x = 3$

Piden: $HE = x\sqrt{2} = (3)\sqrt{2}$

$\therefore HE = 3\sqrt{2}$

Clave D

14.



MNSR es trapecio $\Rightarrow \overline{MN} \parallel \overline{RS}$

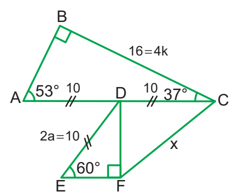
En el ◺MHR (notable de 45°): $a = 4$

En el ◺SAN (notable de 37° y 53°): $b = 3$

$\therefore MN = 4 + 2 + 3 = 9$

Clave C

15.



En el ◺ABC notable de 37° y 53°:

$16 = 4k \Rightarrow k = 4$

$\Rightarrow AC = 5k = 5(4) = 20$

$\Rightarrow AD = DC = 10$

En el ◺EFD notable de 30° y 60°:

$2a = 10$

$a = 5 \Rightarrow DF = 5\sqrt{3}$

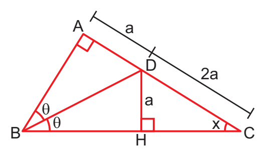
En el ◺FDC:

$x^2 = 10^2 + (5\sqrt{3})^2$

$x = 5\sqrt{7}$

Clave C

16.

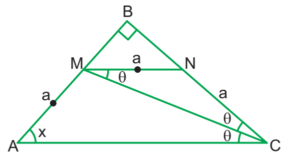


Trazamos $\overline{DH} = \overline{BC} \Rightarrow AD = DH = a$

En el ◺ DHC notable: $HD = \dfrac{DC}{2} \Rightarrow x = 30°$

Clave D

17.



Por dato: $\overline{MN} // \overline{AC}$

$\Rightarrow m\angle NMC = \theta \ \wedge \ MN = NC = a$

Se observa que AMNC es un trapecio isósceles, entonces: $x = 2\theta$

En ⊿ el ABC:

$x + 2\theta = 90° \Rightarrow x + x = 90° \ \Rightarrow x = 45°$

Clave C

Nivel 2 (página 24) Unidad 1

Comunicación matemática

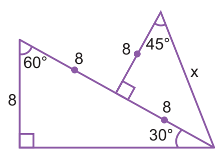
18.

19.

20.

Razonamiento y demostración

21.

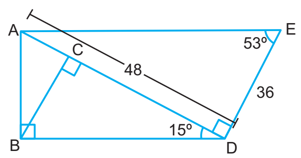


Por triángulo notable: $k = 8$

Piden x: $x = k\sqrt{2} \Rightarrow x = 8\sqrt{2}$

Clave C

22.

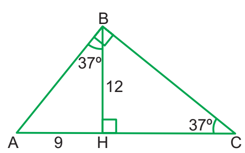


Por propiedad:

$BC = \dfrac{AD}{4} \Rightarrow BC = \dfrac{48}{4} \quad \therefore BC = 12$ m

Clave E

23.

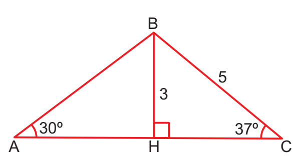


Del gráfico: $\dfrac{HC}{BH} = \dfrac{4}{3} \Rightarrow \dfrac{HC}{12} = \dfrac{4}{3}$

$\therefore$ HC = 16 m

Clave B

24.

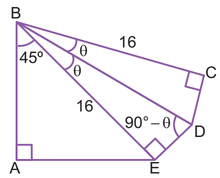


Trazamos $\overline{BH} \perp \overline{AC}$, luego: BH = 3

AB = 3(2) $\quad \therefore$ AB = 6 m

Clave A

25.



Del gráfico:

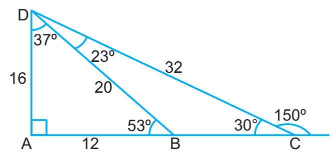
$AE = \dfrac{16}{\sqrt{2}}$

$\therefore AE = 8\sqrt{2}$ m

Clave A

Resolución de problemas

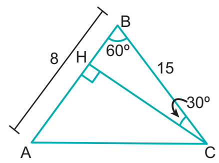
26.



$2p_{\Delta DAB} = 16 + 12 + 20 = 48$

Clave D

27.



Del gráfico:

$BH = \dfrac{15}{2}$

$AH = 8 - \dfrac{15}{2} = \dfrac{1}{2}$

$HC = \dfrac{15}{2}\sqrt{3}$

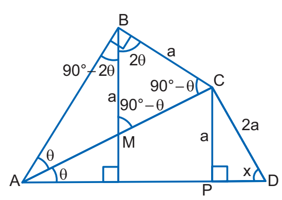
Por el teorema de Pitágoras:

$AC^2 = \left(\dfrac{15}{2}\sqrt{3}\right)^2 + \left(\dfrac{1}{2}\right)^2$

$AC^2 = 169 \qquad \therefore \quad AC = 13$

Clave D

28.



Sea: BM = a

Luego se observa que:

BM = BC (el triángulo BMC es isósceles)

Además: BC = CP ($\overline{AC}$ es bisectriz)

Del triángulo ⊿CPD: notable de 30° y 60°

$\therefore \ x = 30°$

Clave C

Nivel 3 (página 25) Unidad 1

Comunicación matemática

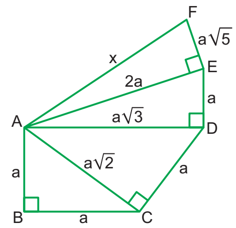
29.

30.

31.

Razonamiento y demostración

32.



Empleando el teorema de Pitágoras en los triángulos rectángulos ABC, ACD, ADE, se calcula el valor de sus correspondientes hipotenusas.

Luego en el ⊿AEF, por el teorema de Pitágoras:

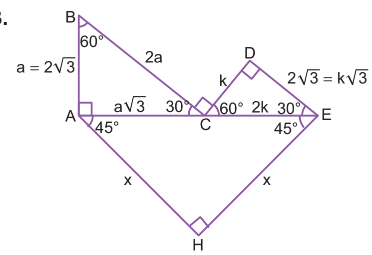
$x^2 = (a\sqrt{5})^2 + (2a)^2 \Rightarrow x^2 = 5a^2 + 4a^2$

$x^2 = 9a^2$

$\therefore x = 3a$

Clave C

33.



Del gráfico:

$a = 2\sqrt{3} \ \wedge \ k\sqrt{3} = 2\sqrt{3} \ \Rightarrow k = 2$

Luego del ⊿AHE notable de 45°: $AE = x\sqrt{2}$

$(a\sqrt{3} + 2k) = x\sqrt{2}$

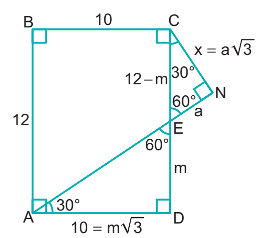
$(2\sqrt{3})\sqrt{3} + 2(2) = x\sqrt{2}$

$(6 + 4) = x\sqrt{2}$

$x\sqrt{2} = 10 \qquad \therefore \ x = 5\sqrt{2}$

Clave A

34.



En el ⊿ADE:

$m\sqrt{3} = 10 \Rightarrow m = \dfrac{10\sqrt{3}}{3}$

En el ⊿ENC:

$CN = a\sqrt{3} \Rightarrow x = a\sqrt{3}$ ...(1)

$CE = 2a \Rightarrow 12 - m = 2a$ ...(2)

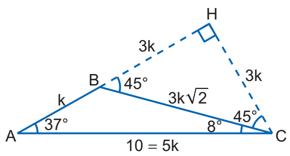
Dividiendo (1) y (2):

$\dfrac{x}{12 - m} = \dfrac{\sqrt{3}}{2} \Rightarrow x = \dfrac{\sqrt{3}}{2}(12 - m)$

$\Rightarrow x = \dfrac{\sqrt{3}}{2}\left(12 - \dfrac{10\sqrt{3}}{3}\right) \quad \therefore \ x = 6\sqrt{3} - 5$

Clave B

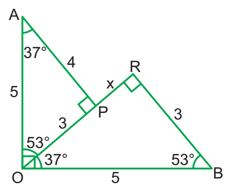
35.



Prolongamos $\overline{AB}$ y trazamos la perpendicular $\overline{CH}$.
Del gráfico: $5k = 10 \Rightarrow k = 2$
Piden: $BC = 3k\sqrt{2} = 3(2)\sqrt{2}$
$\therefore BC = 6\sqrt{2}$

Clave C

36.

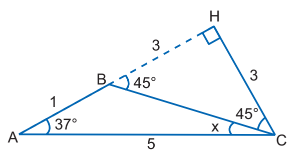


El $\triangle$ORB es notable de 37° y 53°: OR = 4
Del $\triangle$OPA: OP = 3
Piden: PR = x
Del gráfico: $3 + x = OR$
$3 + x = 4$
$\therefore x = 1$

Clave A

Resolución de problemas

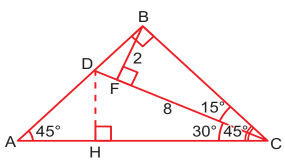
37.



Por ángulo exterior: $37° + m\angle ACB = 45°$
$\therefore m\angle ACB = 8°$

Clave A

38.



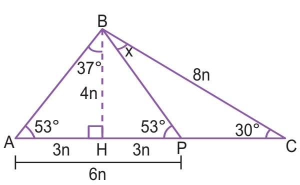
Por propiedad:
$DC = 4BF = 4(2) = 8$

Luego: $DH = \dfrac{8}{2} = 4$

$AD = 4(\sqrt{2})$ $\quad\therefore AD = 4\sqrt{2}$

Clave C

39.

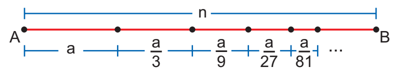


Por ángulo exterior:
$x + 30° = 53°$ $\quad\therefore x = 23°$

Clave B

MARATÓN MATEMÁTICA (página 26)

1.



Hallando a:

$$a + \frac{a}{3} + \frac{a}{9} + \frac{a}{27} + \frac{a}{81} + ... = n$$

$$a\Big[\underbrace{1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + ...}_{\frac{n}{3a}}\Big] = n$$

$$\Rightarrow a + \frac{n}{3} = n \Rightarrow a = \frac{2}{3}n$$

Hallando la suma de los segmentos de orden par:

$$\frac{a}{3} + \frac{a}{27} + \frac{a}{243} + \frac{a}{2187} + ... = S$$

$$a\left[\frac{1}{3} + \frac{1}{27} + \frac{1}{243} + \frac{1}{2187} + ...\right] = S$$

$$a\Big[3 + \underbrace{\frac{1}{3} + \frac{1}{27} + \frac{1}{243} + ...}_{\frac{S}{a}}\Big] = 9S$$

$$\Rightarrow 3a + S = 9S \Rightarrow S = \frac{3a}{8}$$

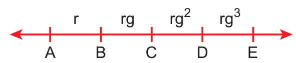
La enésima parte de S:

$$S = \frac{3}{8}\left(\frac{2}{3}n\right) = \frac{n}{4}$$

$$\therefore \frac{\frac{n}{4}}{n} = \frac{1}{4}$$

Clave C

2.



Si AB = r y la razón geométrica: g
$\Rightarrow BC = rg;\ CD = rg^2;\ DE = rg^3$

Por dato; AE = 6
$\Rightarrow AB + BC + CD + DE = 6$
$r + rg + rg^2 + rg^3 = 6$
$r(1 + g + g^2 + g^3) = 6$
$r(1 + g)(1 + g^2) = 6$ ... (α)

Nos piden: $\dfrac{(AC)(AB + CD)}{AB}$

Reemplazando:

$$\Rightarrow \frac{(r + rg)(r + rg^2)}{r} = \frac{r^2(1 + g)(1 + g^2)}{r}$$
$$= r(1 + g)(1 + g^2)$$

Igualando en α:

$$\therefore \frac{(AC)(AB + CD)}{AB} = 6$$

Clave B

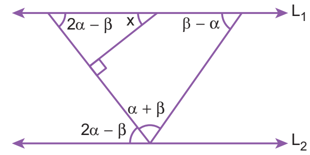
3. Sea α la medida del ángulo:
$S[S_{(\alpha)} - CS_{(\alpha)}] = C[CC_{(\alpha)} - S_{(\alpha)}]$
$180° - [180° - \alpha - (90° - (180° - \alpha))]$
$= 90 - [\alpha - (180° - \alpha)]$
$180° - [270° - 2\alpha] = 90° - 2\alpha + 180°$
$4\alpha = 360°$
$\alpha = 90°$

Nos piden: $C_{\left(\frac{\alpha}{2}\right)}$

$\therefore C_{\left(\frac{\alpha}{2}\right)} = C_{45°} = 90° - 45° = 45°$

Clave D

4.



De la gráfica:
$2\alpha - \beta + \alpha + \beta + \beta - \alpha = 180°$
$2\alpha + \beta = 180°$ ... (I)

$2\alpha - \beta + x = 90°$ ... (II)

Los ángulos son positivos:

$$\left.\begin{array}{l} 2\alpha - \beta > 0 \Rightarrow 2\alpha > \beta \\ \beta - \alpha > 0 \Rightarrow \beta > \alpha \end{array}\right\} \quad 2\alpha > \beta > \alpha$$

$\Rightarrow$ Si α es el mínimo valor entero y de (I):
$2\alpha > \beta > \alpha$
$2\alpha + 2\alpha > 2\alpha + \beta > \alpha + 2\alpha$
$4\alpha > 180° > 3\alpha$

Para $\alpha = 45°$:
$180° > 180° > 135°$ (Falso)

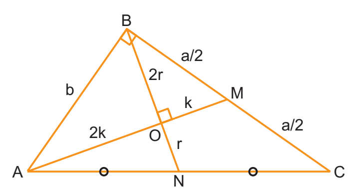
Para $\alpha = 46°$:
$184° > 180° > 138°$ (Verdadero) **mínimo**

Para $\alpha = 47°$:
$188° > 180° > 141°$ (Verdadero)
$\alpha = 46°$, en (I)
$2(46°) + \beta = 180° \Rightarrow \beta = 88°$

En (II).
$2\alpha - \beta + x = 90°$
$2(46°) - 88° + x = 90° \Rightarrow 4° + x = 90°$
$x = 86°$

Clave B

5.



Por el teorema de Pitágoras en:

- $\triangle$BOA: $b^2 = 4k^2 + 4r^2$ ... (α)
- $\triangle$BOM: $\dfrac{a^2}{4} = 4r^2 + k^2$ ... (β)
- $\triangle$ABM: $9k^2 = b^2 + \dfrac{a^2}{4}$ ... (γ)

$(\alpha) - (\beta)$:

$$\left.\begin{array}{l} b^2 = 4k^2 + 4r^2 \\ \dfrac{a^2}{4} = 4r^2 + k^2 \end{array}\right\} \ b^2 - \frac{a^2}{4} = 3k^2 \quad ... (\theta)$$

Igualando: $(\gamma) = 3(\theta)$

$$\Rightarrow b^2 + \frac{a^2}{4} = 3b^2 - \frac{3a^2}{4}$$

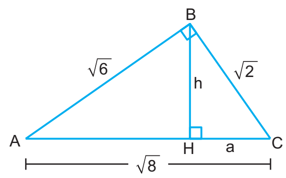
$$a^2 = 2b^2 \Rightarrow b = \frac{\sqrt{2}}{2}a$$

Clave A

6. Los catetos de los triángulos cumplen la relación del teorema de Pitágoras:

$$(\sqrt{8})^2 = (\sqrt{6})^2 + (\sqrt{2})^2$$

Hallando h:



Del $\triangle$AHB: $(\sqrt{6})^2 = h^2 + (\sqrt{8} - a)^2$ ... (α)

Del $\triangle$BHC: $(\sqrt{2})^2 = h^2 + a^2$ ... (β)

$\Rightarrow (\alpha) - (\beta)$:

$$6 = h^2 + 8 + a^2 - 2a\sqrt{8}$$
$$2 = h^2 + a^2 \quad (-)$$

$$4 = 8 - 2a\sqrt{8}$$

$$a = \frac{\sqrt{2}}{2}$$

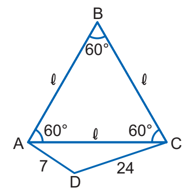
$\therefore$ en (β):

$$\sqrt{2}^2 = h^2 + a^2$$

$$\Rightarrow h = \frac{\sqrt{6}}{2}u$$

Clave D

7.



Si: $m\angle ADC > 90°$

$\Rightarrow \ell^2 > 7^2 + 29^2$

$\ell > 25$

Si P es el perímetro$> \frac{p}{2} = \frac{3\ell}{2}$

$\ell > 25 \Rightarrow \frac{3\ell}{2} > 37{,}5$

$\Rightarrow \frac{p}{2} = \frac{3\ell}{2} > 37{,}5$

$\therefore$ El menor semiperímetro entero es: 38

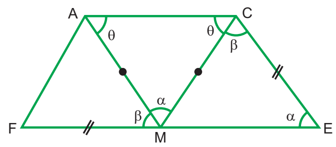
Clave C

Unidad 2

CONGRUENCIA DE TRIÁNGULOS

APLICAMOS LO APRENDIDO (página 28) Unidad 2

1. De la figura:



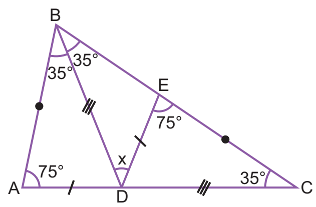
Entonces: $\Delta AMF \cong \Delta MCE$ (caso LAL)

Luego: AF = EM

$\therefore \dfrac{AF}{EM} = 1$

Clave A

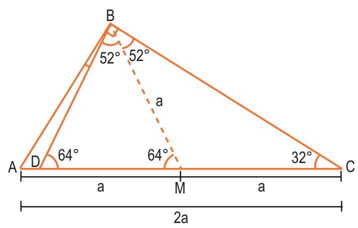
2. De la figura:



$\therefore\ x = 75° - 35° = 40°$

Clave A

3.



Tenemos BM: mediana relativa a la Hipotenusa.

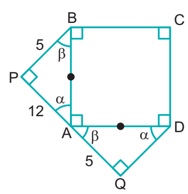
AM = BM = MC = a

Luego: ΔDBM (Isósceles) $\Rightarrow$ BD = BM = 6

a = 6 $\Rightarrow$ x = 2(6) = 12

Clave A

4. De la figura:



El $⊿BPA \cong ⊿AQD$ (caso ALA) por teorema de Pitágoras:

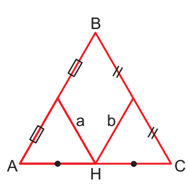
$BA = \sqrt{5^2 + 12^2}$

BA = 13

Piden el perímetro del cuadrado ABCD: 4(13) = 52

Clave A

5.



Dato: AB = 10 $\wedge$ BC = 12

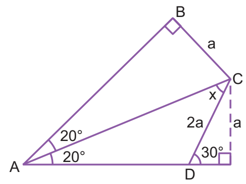
b es base media del $\Delta BCA \Rightarrow$ b = 5

a es base media del $\Delta BAC \Rightarrow$ a = 6

$\therefore$ a + b = 11

Clave E

6.



Dato: CD = 2BC

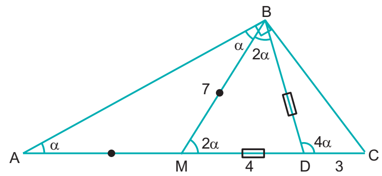
Por ángulo exterior:

20° + x = 30°

$\therefore$ x = 10°

Clave B

7.



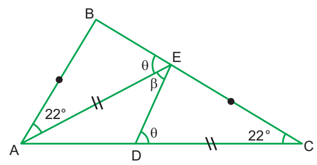
Se traza la mediana BM, entonces:

AM = MB = MC

$\therefore$ BD = 4

Clave B

8.



Del gráfico se observa que:

$\Delta BAE \cong \Delta ECD$ (caso LAL)

$\Rightarrow m\angle CDE = m\angle AEB = \theta$

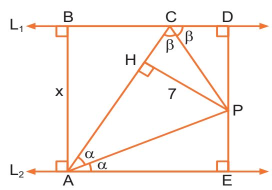
En el ΔDCE:

$\theta + 22° = \theta + \beta$

$\therefore \beta = 22°$

Clave C

9.



Trazamos $\overline{PD}$ perpendicular a $\overleftrightarrow{L_1}$.

Trazamos $\overline{PE}$ perpendicular a $\overleftrightarrow{L_2}$.

Por el teorema de la bisectriz de un ángulo:

DP = PH = 7 $\wedge$ PE = PH = 7

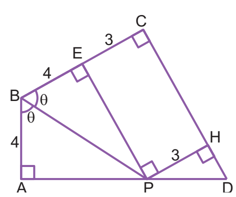
Luego: x = DP + PE

x = 7 + 7

$\therefore$ x = 14

Clave B

10.



Trazamos $\overline{PE} \perp \overline{BC}$.

Por el teorema de la bisectriz de un ángulo:

BE = AB = 4

Del gráfico: EC = PH $\Rightarrow$ EC = 3

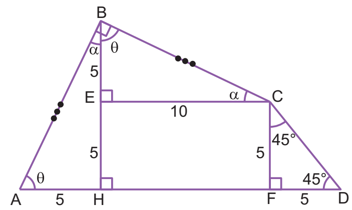
Piden:

BC = BE + EC = 4 + 3

$\therefore$ BC = 7

Clave C

11.



Trazamos $\overline{CE} \perp \overline{BH}$ y $\overline{CF} \perp \overline{AD}$

Se observa que:

$⊿AHB \cong ⊿BEC$ (caso ALA)

$\Rightarrow$ BE = AH = 5 $\wedge$ EC = BH = 10

Como: BH = BE + EH

10 = 5 + EH $\Rightarrow$ EH = 5

Del gráfico:

CF = EH = 5

Del $⊿CFD$ notable de 45°:

CF = FD = 5

Entonces:

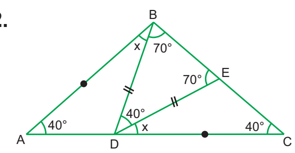
AD = AH + HF + FD

AD = 5 + 10 + 5

$\therefore$ AD = 20

Clave E

12.



Por dato: AB = CD

En el ΔBDE : $m\Delta BED = 70°$

$\Delta ABD \cong \Delta CDE$ (caso LAL)

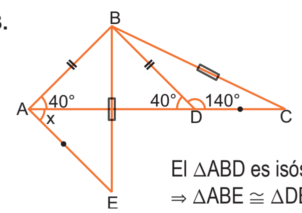
$\Rightarrow m\angle ECD = 40°$

Del gráfico: 40° + x = 70°

$\therefore$ x = 30°

Clave D

13.



El ΔABD es isósceles.

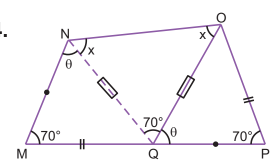
$\Rightarrow \Delta ABE \cong \Delta DBC$ (caso LLL)

$\Rightarrow$ x + 40° = 140°

$\therefore$ x = 100°

Clave B

14.



$\Delta NMQ \cong \Delta QPO$ (caso LAL)

Entonces: NQ = QO

$m\angle MNQ = m\angle OQP = \theta$

Por ángulo exterior:

$m\angle NQO = 70°$

En el ΔNQO isósceles:

70° + 2x = 180°

$\therefore$ x = 55°

Clave E

PRACTIQUEMOS
Nivel 1 (página 30) Unidad 2

Comunicación matemática

1.

2.

3.

Razonamiento y demostración

4. Por propiedad de la mediatriz:
$\therefore x = 8$

Clave D

5. Propiedad de la base media:
$\therefore x = \dfrac{8\sqrt{2}}{2} = 4\sqrt{2}$

Clave B

6. Por congruencia de triángulos (caso LAL):
$a = 14;\ b = 16 \Rightarrow a + b = 30$

Clave C

7. Por congruencia (caso LAL):
$\therefore \alpha = 37°$

Clave C

8. Por propiedad de la base media:
$x - 4 = 6 \qquad \therefore x = 10$

Clave B

9. Por propiedad de base media:
$4 = \dfrac{2x - 12}{2} \Rightarrow 2x - 12 = 8$
$2x = 20$
$\therefore x = 10$

Clave A

10. Por congruencia de triángulos:
$2x = 8 \Rightarrow x = 4$

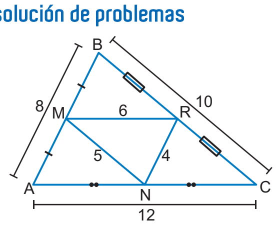
Clave D

11. Por congruencia de triángulos (caso LLL):
$a + 18° = 105°$
$\therefore a = 87°$

Clave A

Resolución de problemas

12.



Por el teorema de los puntos medios:

$MR = \dfrac{AC}{2} = \dfrac{12}{2} = 6$

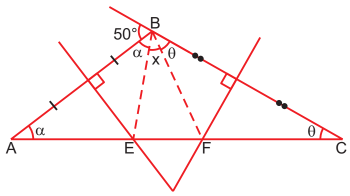
$MN = \dfrac{BC}{2} = \dfrac{10}{2} = 5$

$NR = \dfrac{AB}{2} = \dfrac{8}{2} = 4$

Piden:
$2p_{\triangle MRN} = 4 + 5 + 6$
$\therefore\ 2p_{\triangle MRN} = 15$

Clave B

13.



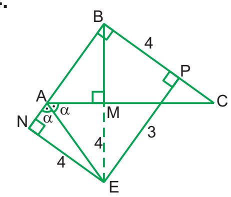
Del $\triangle ABC$:
$\alpha + \theta = 50° \qquad ...(1)$

Del gráfico:
$x + \alpha + \theta + 50° = 180°$
$x + \alpha + \theta = 130° \qquad ...(2)$

Reemplazando (1) en (2):
$x + 50° = 130°$
$\therefore x = 80°$

Clave C

14.

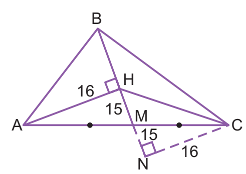


Por el teorema de la bisectriz:
$ME = NE = 4$

Luego en el ⊿BPE,
por el teorema de Pitágoras:
$(BE)^2 = 4^2 + 3^2$
$(BE)^2 = 25$
$\therefore BE = 5$

Clave C

15.



⊿CMN ≅ ⊿AHM (caso ALA)

Por el teorema de Pitágoras:
$(HC)^2 = (30)^2 + 16^2$
$(HC)^2 = 1156 \Rightarrow HC = 34$

Clave A

Nivel 2 (página 31) Unidad 2

Comunicación matemática

16.

17.

18.

Razonamiento y demostración

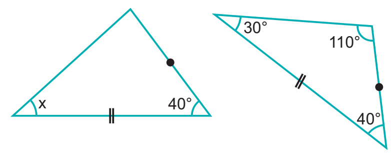
19. Se deduce que: $m = 12$
$\therefore 2m - 5 = 2(12) - 5 = 19$

Clave E

20. De los triángulos congruentes (caso LAL): $x = 8$
$\therefore \sqrt{3x + 1} = \sqrt{3(8) + 1} = 5$

Clave B

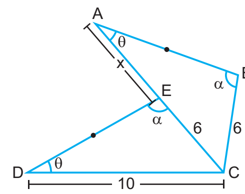
21. Los triángulos son congruentes (caso LAL):



$\therefore x = 30°$

Clave D

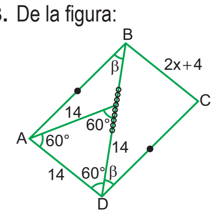
22.



$\triangle ABC \cong \triangle DEC$ (caso ALA)
$\Rightarrow EC = 6 \wedge AC = 10$
$\Rightarrow AE = AC - EC$
$\therefore AE = 10 - 6 = 4$

Clave D

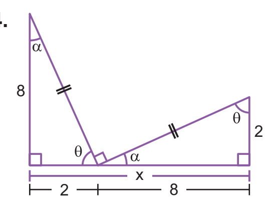
23. De la figura:



$\triangle ABD \cong \triangle CDB$
(caso LAL)
$\Rightarrow\ 2x + 4 = 14$
$2x = 10$
$\therefore\ x = 5$

Clave B

24.

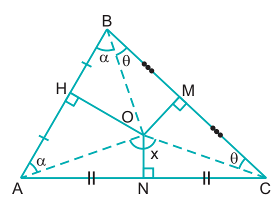


De los triángulos congruentes (caso ALA):
$x = 2 + 8 = 10$

Clave C

Resolución de problemas

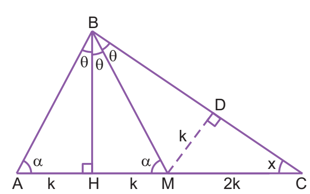
25.



Piden: $m\angle AOC = x$
Por dato: $m\angle ABC = 40°$
$\Rightarrow \alpha + \theta = 40°$
Por propiedad: $x = 2\alpha + 2\theta$
$x = 2(\alpha + \theta) = 2(40°)$
$\therefore x = 80°$

Clave B

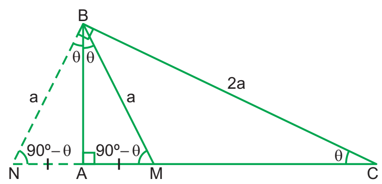
26.



El $\triangle ABM$ es isósceles, entonces: $AH = HM$
Por el teorema de la bisectriz: $HM = MD$
Luego, el ⊿MDC resulta notable de 30° y 60°.
$\therefore x = 30°$

Clave C

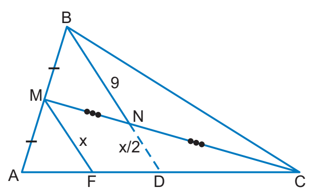
27.



Piden: $m\angle BCA = \theta$
Prolongamos $\overline{CA}$ hasta N, tal que: $AM = AN$
Entonces: $BM = BN = a$
Luego, el ⊿NBC resulta notable de 53°/2.
$\therefore \theta = 53°/2$

Clave B

28.



Por dato: $\overline{MF} \,//\, \overline{BN}$

Por el teorema de los puntos medios:

$$ND = \frac{MF}{2} = \frac{x}{2}$$

$$MF = \frac{BD}{2} \Rightarrow x = \frac{9 + \frac{x}{2}}{2}$$

$$2x = 9 + \frac{x}{2}$$

$$\frac{3x}{2} = 9$$

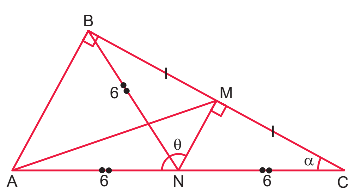
$$\therefore x = 6$$

Clave A

Nivel 3 (página 32) Unidad 2

Comunicación matemática

29.



Sean las medianas: $\overline{BN}$ y $\overline{AM}$
Por propiedad en el ⊿ABC: $BN = AN = NC$
Luego: $\theta = 90° + \alpha \Rightarrow \theta$ es obtuso.
Entonces el $\triangle ANM$ es obtusángulo, además
Si $AM > AN \Rightarrow AM > BN$

Análogamente se demuestra que la otra mediana que parte de C es mayor que BN.
Por lo tanto, la menor mediana es BN y mide 6.

Clave C

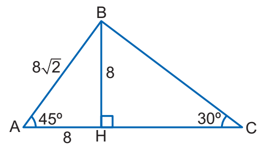
30.

31. Los triángulos ABC y ADC son isósceles.

$$\therefore \left.\begin{cases} 8 + 5 = 13 \\ 3 + 2 = 5 \end{cases}\right\} 18$$

Razonamiento y demostración

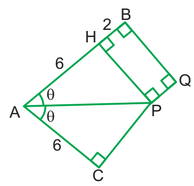
32.



Trazamos $\overline{BH} \perp \overline{AC}$, luego:
El ⊿ABH es notable de 45°
Si $AB = 8\sqrt{2} \Rightarrow BC = 8(2)$
$\therefore BC = 16$

Clave B

33.

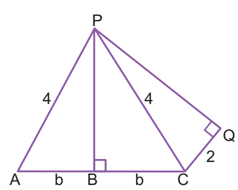


Por el teorema de la bisectriz:
$AC = AH$

Luego: $HB = PQ$
$\therefore PQ = 2$

Clave A

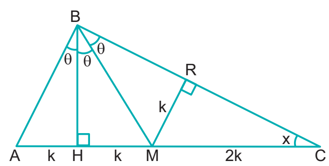
34.



Por el teorema de Pitágoras:
$PQ^2 = 4^2 - 2^2$
$PQ^2 = 12$
$\therefore PQ = 2\sqrt{3}$

Clave D

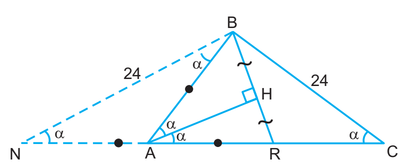
35.



El ⊿MRC notable de 30° y 60°.
$\therefore x = 30°$

Clave D

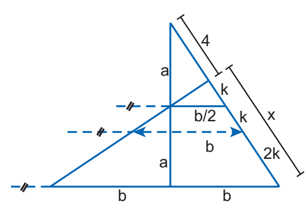
36.



$\overline{AH}$ es base media: $\Rightarrow AH = \dfrac{24}{2} = 12$

Clave C

37.



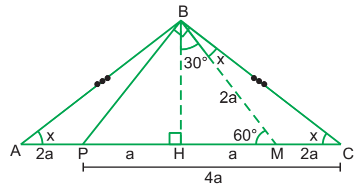
Del gráfico:
$4 + k = 3k$
$4 = 2k$
$2 = k$

Luego:
$x = 4k$
$\therefore x = 8$

Clave B

Resolución de problemas

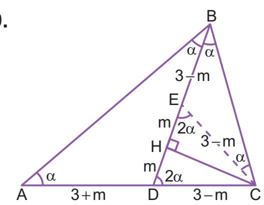
38.



Sea: $AP = 2a$
Se traza $\overline{BH}$, altura del $\triangle ABC$.
Se traza $\overline{BM}$, mediana del ⊿PBC.
Luego, el ⊿BHM resulta notable de 30° y 60°.
En el $\triangle BMC$: $\quad x + x = 60°$
$2x = 60°$
$\therefore x = 30°$

Clave A

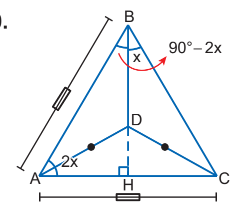
39.



$\triangle ADB$: isósceles $\Rightarrow AD = 3 + m$
Se traza $CE = CD$, $\triangle DCE$ es isósceles
$\triangle BEC$ es isósceles $\Rightarrow BE = EC = DC$
$\therefore AC = 6$

Clave A

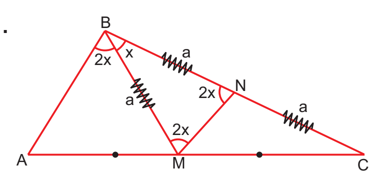
40.



Por dato:
$AB = AC \Rightarrow m\angle B = m\angle C = 90° - x$
Prolongamos $\overline{BD}$:
$m\angle ABD = 90° - 2x$, entonces, $m\angle AHB = 90°$
$\overline{BH}$ es mediatriz, entonces: $\quad 90° - 2x = x$
$3x = 90°$
$x = 30°$

Clave C

41.



Se traza $\overline{MN} \,//\, \overline{AB}$, y por el teorema de los puntos medios tenemos: $BN = NC$
En el triángulo isósceles MBN:
$x + 2x + 2x = 180° \Rightarrow x = 36°$

Clave E

PRACTIQUEMOS

Nivel 1 (página 36) Unidad 2

Comunicación matemática

1.

2.

Razonamiento y demostración

3. Piden: nombre del polígono

Dato: D = 2n

Entonces:

$$\frac{n(n-3)}{2} = 2n$$

$$n - 3 = 4$$

$\Rightarrow n = 7 \quad \therefore$ Es un heptágono.

Clave C

4. $S_{m\angle i} = 180°(5-2)$

$S_{m\angle i} = 540°$

Clave A

5. $S_{m\angle e} = 360°$

Clave A

6. $D_T = \frac{8}{2}(8-3)$

$D_T = 4(5)$

$D_T = 20$

Clave A

7. $S_{m\angle i} = 180°(15-2)$

$S_{m\angle i} = 180°(13)$

$S_{m\angle i} = 2340°$

Clave C

8. $D_T = \frac{20}{2}(20-3)$

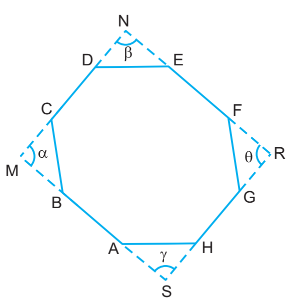
$D_T = 10(17)$

$D_T = 170$

Clave E

Resolución de problemas

9.



Las prolongaciones de los lados del octágono forman el cuadrilátero MNRS, en donde:

$$S_{m\angle i} = \alpha + \beta + \theta + \gamma$$

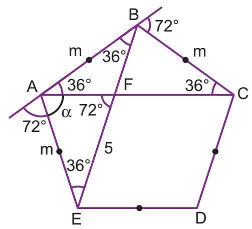
$$180°(4-2) = \alpha + \beta + \theta + \gamma$$

$$180°(2) = \alpha + \beta + \theta + \gamma$$

$\therefore \alpha + \beta + \theta + \gamma = 360°$

Clave D

10.



Sea la medida del lado del polígono: m

Por ser un polígono regular:

$m\angle e = \frac{360°}{5} = 72°$

En el ΔAFE: $\alpha + 72° + 36° = 180°$

$\Rightarrow \alpha = 72°$

Entonces el ΔAFE resulta isósceles:

AE = EF

$\therefore m = 5$

Clave D

11. Sea n el número de lados del polígono.

Por dato:

$S_{m\angle i}$ + n.° de vértices + n.° de lados = 3280

$180(n-2) + n + n = 3280$

$180n - 360 + 2n = 3280$

$182n = 3640$

$\Rightarrow n = 20$

Por lo tanto, el polígono es un icoságono.

Clave A

12. Sea n el número de lados del polígono equiángulo, entonces:

$$\Rightarrow \frac{n(n-3)}{2} = 35$$

$n(n-3) = 70 = 10(7)$

$n(n-3) = 10(10-3)$

$\Rightarrow n = 10$

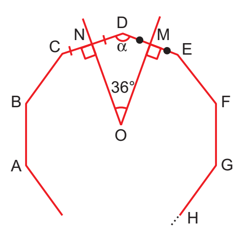
Piden:

$$m\angle i = \frac{180°(n-2)}{n} = \frac{180°(8)}{10} = 144°$$

$\therefore m\angle i = 144°$

Clave B

13.



Por dato: el polígono es equiángulo

Sea: $m\angle i = \alpha$ ...(1)

En el polígono NOMD:

$90° + 36° + 90° + \alpha = 180°(4-2)$

$216° + \alpha = 360°$

$\alpha = 144°$

Reemplazando en (1):

$m\angle i = 144°$

Sea n el número de lados del polígono:

$$\frac{180°(n-2)}{n} = 144° \Rightarrow 5(n-2) = 4n$$

$5n - 10 = 4n$

$\Rightarrow n = 10$

Piden: el n.° de diagonales (D_T)

$$D_T = \frac{n(n-3)}{2} = \frac{10(10-3)}{2} = 35$$

$\therefore D_T = 35$

Clave E

14. Dato: el polígono es regular

n.° de lados	$m\angle e$
n	$\frac{360°}{n}$
n – 6	$\frac{360°}{n-6}$

Del enunciado:

$$\frac{360°}{n-6} = \frac{360°}{n} + 80°$$

Agrupando términos:

$$360°\left(\frac{1}{n-6} - \frac{1}{n}\right) = 80°$$

$$360° . \frac{6}{(n-6)(n)} = 80°$$

$27 = (n-6)n$

$9(3) = n(n-6)$

$9(9-6) = n(n-6)$

$\Rightarrow n = 9$

Por lo tanto, el polígono tiene 9 lados.

Clave E

15. $m_1\angle i - m_2\angle i = 10°$

$$\frac{180°(n-2)}{n} - \frac{180°(n-6-2)}{n-6} = 10°$$

$$\frac{180°(n-2)}{n} - \frac{180°(n-8)}{n-6} = 10°$$

$n^2 - 6n - 216 = 0$

$(n-18)(n+12) = 0$

$n = 18$

Clave D

16. $\frac{n^2}{2} - \frac{n(n-3)}{2} = 6$

$n^2 - n^2 + 3n = 12 \Rightarrow n = 4$

Clave E

Nivel 2 (página 36) Unidad 2

Comunicación matemática

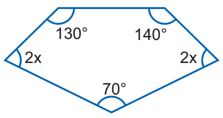
17.

18.

19.

Razonamiento y demostración

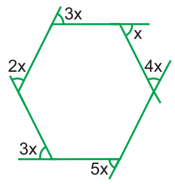
20.



$4x + 340° = 180°(5 – 2)$
$4x = 540° – 340°$
$4x = 200°$
$x = 50°$

Clave E

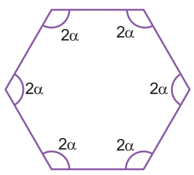
21.



$18x = 360°$
$x = 20°$

Clave A

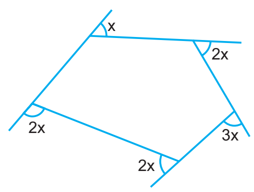
22.



$12\alpha = 180°(6 – 2)$
$12\alpha = 720°$
$\alpha = 60°$

Clave D

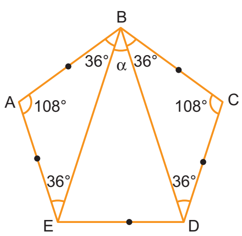
23.



$10x = 360°$
$x = 36°$

Clave C

24.



Se tiene:

$m\angle A = m\angle i = \dfrac{180°(5-2)}{5} = 108°$

En el ΔEAB isósceles:
$m\angle ABE = m\angle AEB = 36°$

En el ΔBCD isósceles:
$m\angle CBD = m\angle CDB = 36°$

Como ABCDE es un polígono regular:
$m\angle A = m\angle B = 108°$

Entonces:
$36° + \alpha + 36° = 108° \Rightarrow \alpha = 36°$

Clave B

Resolución de problemas

25. Piden: número de diagonales

Dato: $S_i – S_e = 720°$

Entonces:
$180°(n – 2) – 360° = 720°$
$180°(n – 2) = 1080°$
$n – 2 = 6$
$\Rightarrow n = 8$

$\therefore\ D_T = \dfrac{n(n-3)}{2} = \dfrac{8(5)}{2} = 20$

Clave E

26. Piden: n

Dato: $D_1 – D_2 = 8$

Lados: 1° $\longrightarrow$ n
2° $\longrightarrow$ (n – 1)

$\Rightarrow \dfrac{n(n-3)}{2} - \dfrac{(n-1)(n-4)}{2} = 8$
$n(n – 3) – (n – 1)(n – 4) = 16$
$n^2 – 3n – (n^2 – 5n + 4) = 16$
$n^2 – 3n – n^2 + 5n – 4 = 16$
$2n = 20$
$\Rightarrow n = 10 \Rightarrow n = 10$ lados

Clave B

27. Piden: nombre del polígono

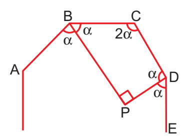
Del enunciado:

$\Rightarrow 180°(5n – 2) = 6[180°(n – 2)]$
$5n – 2 = 6n – 12$
$10 = n$

Por lo tanto, el polígono es un decágono.

Clave B

28.



Sea n el número de lados del polígono ABCDE...

$m\angle i = 2\alpha$

$\dfrac{180°(n-2)}{n} = 2\alpha$ ...(1)

En el polígono BCDP de 4 lados:
$\alpha + 2\alpha + \alpha + 90° = 180°(4 – 2) = 360°$

$\Rightarrow \alpha = \dfrac{135°}{2}$ ...(2)

Reemplazando (2) en (1):

$180°\dfrac{(n-2)}{n} = 135° \Rightarrow 4(n – 2) = 3n$

$\therefore\ n = 8$

Clave D

29. Piden: n

Del enunciado:

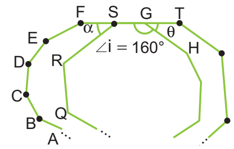
$\dfrac{(n+1)(n-2)}{2} = 6 + \dfrac{n(n-3)}{2}$

$\dfrac{(n+1)(n-2)}{2} = \dfrac{12 + n(n-3)}{2}$

$n^2 – n – 2 = 12 + n^2 – 3n$
$2n = 14$
$\therefore n = 7$

Clave A

30.



Calculamos el ángulo interno del polígono ABCD...

$m\angle i = \dfrac{180°(n-2)}{n} = \dfrac{180°(18-2)}{18} = 160°$

$\Rightarrow 160° + \theta = 180° \Rightarrow \theta = 20°$
Del dato: $\alpha + \theta = 29°$
$\alpha + 20° = 29° \Rightarrow \alpha = 9°$

α es un ángulo externo del polígono QRST...

$\alpha = \dfrac{360°}{n}$

$9° = \dfrac{360°}{n} \Rightarrow n = 40$ lados

Clave D

Nivel 3 (página 37) Unidad 2

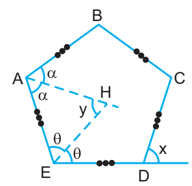
Comunicación matemática

31.

32.

Razonamiento y demostración

33.



Por dato: el polígono es regular

$\Rightarrow m\angle i = \dfrac{180°(5-2)}{5} = 108°$

Del gráfico:
$m\angle i = 2\alpha = 108° \Rightarrow \alpha = 54°$
$m\angle i = 2\theta = 108° \Rightarrow \theta = 54°$

En el ΔAHE:
$\alpha + \theta + y = 180°$
$54° + 54° + y = 180° \Rightarrow y = 72°$

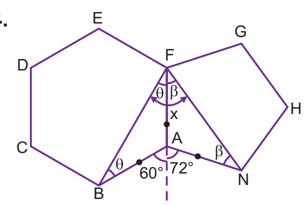
Además: $m\angle e = x$

$\dfrac{360°}{5} = x \Rightarrow x = 72°$

Piden:
$x + y = 72° + 72° \Rightarrow x + y = 144°$

Clave D

34.



Para el polígono regular ABCDEF:

$m\angle e = \dfrac{360°}{6} = 60°$

Además: $2\theta = 60° \Rightarrow \theta = 30°$

Para el polígono regular AFGHN:

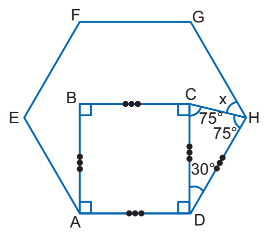
$m\angle e = \dfrac{360°}{5} = 72°$

Además: $2\beta = 72° \Rightarrow \beta = 36°$

Piden:
$x = \theta + \beta$
$x = 30° + 36° \Rightarrow x = 66°$

Clave D

35.



Para el hexágono regular:

$$m\angle i = \frac{180°(6-2)}{6} = 120°$$

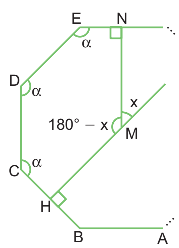
$\Rightarrow m\angle ADH = 120°$
$90° + m\angle CDH = 120° \Rightarrow m\angle CDH = 30°$

El $\triangle CDH$ es isósceles, luego:
$m\angle HCD = m\angle CHD = 75°$

Del gráfico:
$m\angle i = m\angle GHD$
$120° = x + 75° \Rightarrow x = 45°$

Clave B

36.



Por dato, el polígono es un dodecágono equiángulo.
Entonces tiene doce lados ($n = 12$).

Luego:

$$\alpha = m\angle i = \frac{180°(n-2)}{n}$$

$$\alpha = \frac{180°(12-2)}{12} = 150° \Rightarrow \alpha = 150°$$

En el hexágono HCDENM:

$90° + 3\alpha + 90° + 180° - x = S_{m\angle i}$

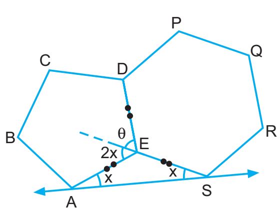
$360° + 3(150°) - x = 180°(6-2)$

$810° - x = 720°$

$\therefore x = 90°$

Clave E

37.



Del gráfico: $\angle\theta$ es el ángulo exterior del hexágono regular

Entonces:

$$\theta = m\angle e = \frac{360°}{6} = 60° \Rightarrow \theta = 60°$$

Luego:
$2x + \theta$: es el ángulo interior del pentágono regular

$$2x + \theta = m\angle i = \frac{180°(5-2)}{5} = 108°$$

$\Rightarrow \quad 2x + 60° = 108°$

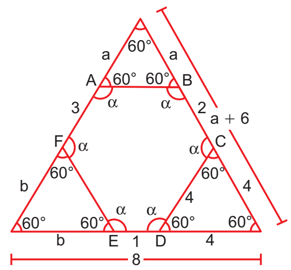
$\therefore x = 24°$

Clave B

Resolución de problemas

38. Piden: $2p$ (perímetro)

Dato: $DE = 1$, $BC = 2$, $AF = 3$ y $CD = 4$



Si es un hexágono equiángulo:

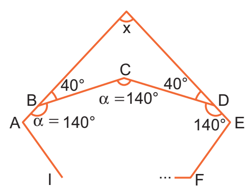
$\Rightarrow 6\alpha = 180°(n-2)$
$6\alpha = 180°(6-2) = 720°$
$\alpha = 120°$

Se prolongan los lados y se forma un triángulo equilátero

$\Rightarrow b = 3 \ \wedge \ a = 2$
$\therefore \ 2p = a + b + 10 = 15$

Clave B

39.



$n = 9$

$$\alpha = m\angle i = \frac{180°(n-2)}{n} = \frac{180°(7)}{9} = 140°$$

$\Rightarrow 140° = 40° + x + 40°$
$140° = 80° + x \Rightarrow x = 60°$

Clave E

40. Sea n el número de lados del polígono.

Del enunciado:

$$2\left(\frac{n(n-3)}{2}\right) = \frac{(n+3)(n)}{2}$$

$$2n - 6 = n + 3$$

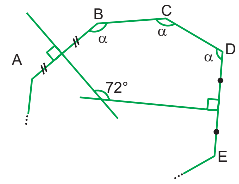
$$n = 9$$

$\Rightarrow \ S m\angle i = 180°(9-2)$

$\therefore S m\angle i = 1260°$

Clave D

41.



Del gráfico:
$3\alpha + 252° = 180°(6-2)$
$3\alpha = 468°$
$\alpha = 156°$

Luego:

$$\frac{180°(n-2)}{n} = 156°$$

$15n - 30 = 13n \Rightarrow n = 15$

Luego:

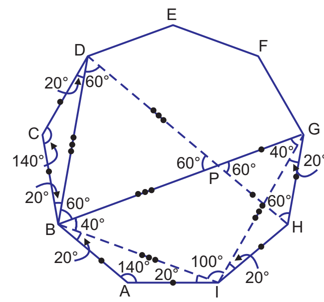
$$D_T = \frac{n(n-3)}{2}$$

$$D_T = \frac{15(15-3)}{2} = \frac{15(12)}{2}$$

$\therefore D_T = 90$

Clave B

42.

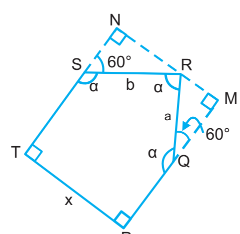


El $\triangle BPD$ y el $\triangle PGH$ resultan equiláteros.
$BD = BP$ y $AB = GH = PG$

Dato:
$AB + BD = 4$
$PG + BP = 4 \Rightarrow BG = 4$

Clave C

43.



Del pentágono PQRST:
$3\alpha + 180° = 540°$
$3\alpha = 360°$
$\alpha = 120°$

Del gráfico:
$NM = PT$

Luego:

$NR = \frac{b}{2}\sqrt{3}$

$RM = \frac{a}{2}\sqrt{3}$

Entonces:

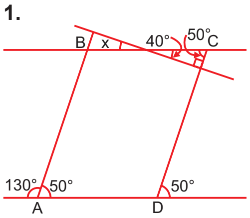
$NM = \left(\frac{a+b}{2}\right)\sqrt{3}$

$\therefore \ PT = \left(\frac{a+b}{2}\right)\sqrt{3}$

Clave D

APLICAMOS LO APRENDIDO (página 39) Unidad 2

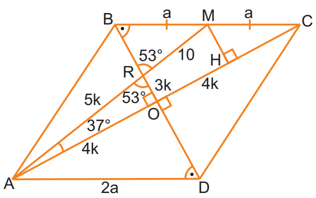
1.



ABCD paralelogramo,
m∠BCD = 50°
∴ x = 40°

Clave C

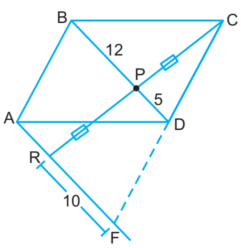
2.



Piden: BD
En el gráfico:
$\triangle ARD \sim \triangle BRM$
$\frac{5k}{10} = \frac{2a}{a} \Rightarrow k = 4$
Trazamos $\overline{MH} \perp \overline{AC}$
⊿AHM (notable 37° y 53°)
Como AM = 5k + 10 = 30 ⇒ MH = 18
Además $\overline{MH}$ es base media del ⊿BOC.
Luego: BO = 36
Entonces: BD = 2(36) = 72

Clave E

3. Piden: AR

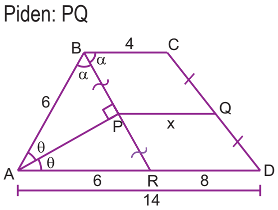


Dato: $\overline{BD} \parallel \overline{AF}$
En el △RCF, $\overline{PD}$ es base media:
RF = 2(PD)
⇒ RF = 10

Luego en el paralelogramo ABDF: AF = BD
Entonces: AR + 10 = 17 ⇒ AR = 7

Clave C

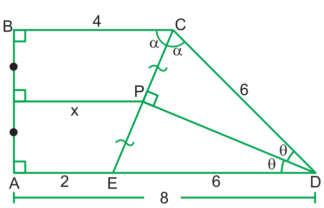
4. Piden: PQ



En el trapecio BCDR: $x = \frac{8+4}{2} = 6$

Clave B

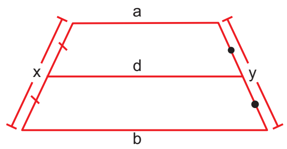
5.



Piden: x
Del trapecio ABCE:
$x = \frac{4+2}{2}$ (propiedad de la mediana) ⇒ x = 3

Clave B

6. Sea el trapecio:



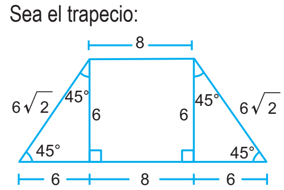
Dato: $\frac{a}{b} = \frac{1k}{5k}$; x + y = 30 m

Sabemos:
Perímetro = x + y + a + b
66 = 30 + k + 5k
36 = 6k
k = 6
Luego la base media:
$d = \frac{b+a}{2} = \frac{6+30}{2}$

$d = \frac{36}{2} = 18$ m

Clave B

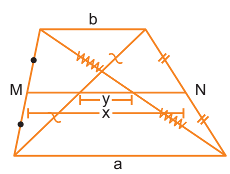
7. Sea el trapecio:



Base media = $\frac{20+8}{2} = \frac{28}{2} = 14$ m

Clave A

8.

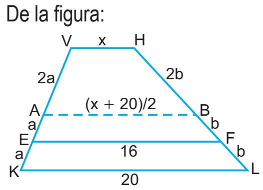


Mediana: $x = \frac{a+b}{2}$

Segmento que une los puntos medios de las diagonales:
$y = \frac{a-b}{2}$
$\Rightarrow x + y = \frac{a+b}{2} + \frac{a-b}{2} = 25 \quad \therefore a = 25$

Clave A

9. De la figura:

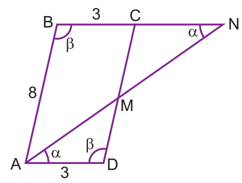


En el ⏢ABLK: $\frac{\frac{(x+20)}{2} + 20}{2} = 16$

Resolviendo: (x + 20)/2 = 32 – 20
(x + 20)/2 = 12
x + 20 = 24 ⇒ x = 4

Clave B

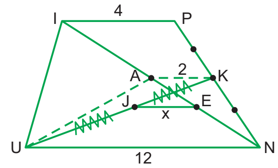
10. De la figura:



$\frac{MD}{8} = \frac{3}{BN}$
∴ (MD)(BN) = 24

Clave E

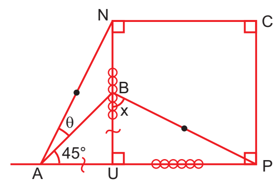
11. De la figura:



Trazo $\overline{AK} \parallel \overline{IP}$
IE = 3EN (dato) ⇒ AE = EN
En el ⏢UAKN: $x = \frac{12-2}{2} = \frac{10}{2} = 5$

Clave C

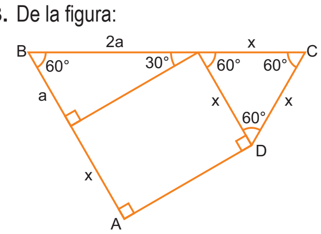
12. De la figura:



⊿AUN ≅ ⊿BUP ⇒ BU = AU
⇒ ∠BAU = 45°
También: x = θ + 45°

Clave D

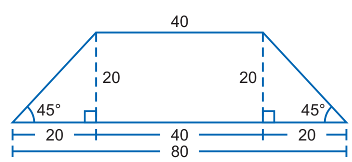
13. De la figura:



Por dato: 2AB – BC = 8
2(a + x) – (2a + x) = 8
2a + 2x – 2a – x = 8 ∴ x = 8

Clave C

14. Según el enunciado:



La mediana es:

$\frac{40+80}{2} = 60$ m

Clave B

PRACTIQUEMOS

Nivel 1 (página 41) Unidad 2

Comunicación matemática

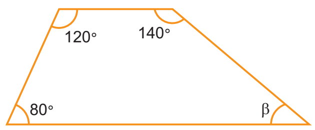
1.

2.

3.

Razonamiento y demostración

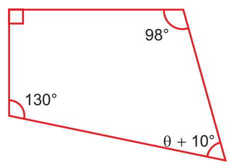
4.



$120° + 80° + 140° + \beta = 360°$
$\beta = 360° - 340°$
$\therefore\ \beta = 20°$

Clave C

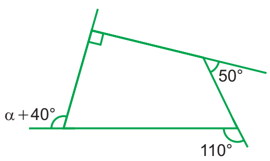
5.



$90° + 98° + 130° + \theta + 10° = 360°$
$\theta = 360° - 328°$
$\therefore\ \theta = 32°$

Clave E

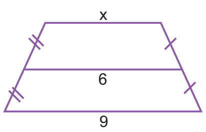
6.



$50° + 90° + 110° + \alpha + 40° = 360°$
$290° + \alpha = 360°$
$\therefore\ \alpha = 70°$

Clave B

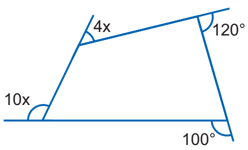
7.



$6 = \dfrac{9 + x}{2}$
$12 = 9 + x \Rightarrow x = 3$

Clave A

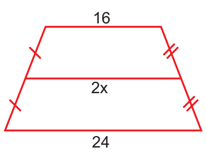
8.



$220° + 14x = 360°$
$14x = 140° \Rightarrow x = 10°$

Clave B

9.



$2x = \dfrac{16 + 24}{2}$
$2x = \dfrac{40}{2}$
$2x = 20 \Rightarrow x = 10$

Clave C

Resolución de problemas

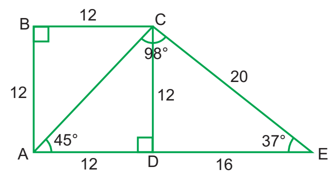
10. Sean los lados: 2x; 9x
$\Rightarrow 2x + 9x + 2x + 9x = 88$
$22x = 88 \Rightarrow x = 4$
Los lados son: 8; 36

Clave E

11. Sean los ángulos: 2a; 7a
$\therefore 2a + 7a = 180°$
$9a = 180° \Rightarrow a = 20°$
El mayor ángulo es: $7a = 140°$

Clave D

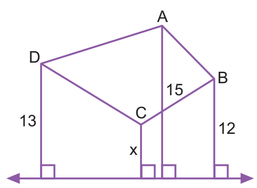
12. Del enunciado:



El ⊿CDE es notable de 37° y 53°
$\Rightarrow CD = 12$ y $DE = 16$
Luego:
Perímetro de ABCD: $4 \times 12 = 48$ m

Clave E

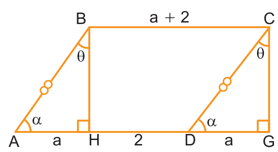
13. De acuerdo con el enunciado:



Por propiedad:
$15 + x = 13 + 12 \Rightarrow x = 10$

Clave E

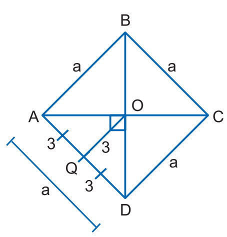
14.



Dato:
AG = 10
El ⊿AHB ≅ DGC (caso ALA)
Entonces:
$2a + 2 = 10 \Rightarrow a = 4$
Perímetro del rombo ABCD = 4(a + 2)
Perímetro del rombo ABCD = 4(6)
∴ Perímetro del rombo ABCD = 24

Clave C

15.



Del gráfico: a = 6
Perímetro del rombo ABCD = 4(6)
∴ Perímetro del rombo ABCD = 24

Clave A

Nivel 2 (página 42) Unidad 2

Comunicación matemática

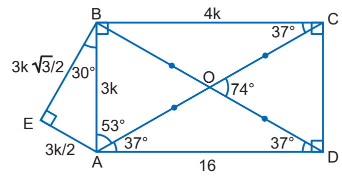
16.

17.

18.

Razonamiento y demostración

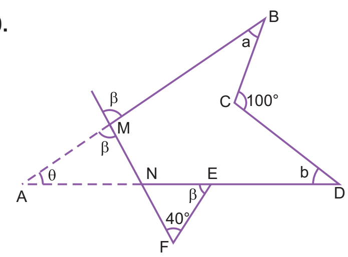
19.



Por dato: ABCD es un rectángulo
Del gráfico: $4k = 16$
$\Rightarrow k = 4$
Piden:
$EB = \dfrac{3k\sqrt{3}}{2} = \dfrac{3(4)\sqrt{3}}{2} = 6\sqrt{3}$
$\therefore EB = 6\sqrt{3}$

Clave D

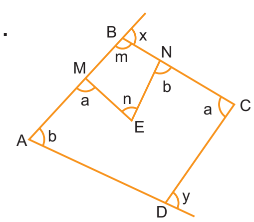
20.



En el gráfico AMNFE por propiedad:
$\beta + \theta = \beta + 40° \Rightarrow \theta = 40°$
En el cuadrilátero no convexo ABCD:
$\theta + a + b = 100°$
$40° + a + b = 100°$
$\therefore a + b = 60°$

Clave E

21.



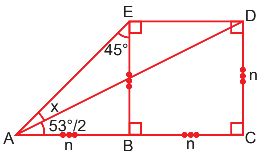
Por dato:
$m + n = 120°$

En el cuadrilátero ABCD por propiedad:
$a + b = x + y$ ...(1)

En el cuadrilátero BMEN por propiedad:
$a + b = m + n$ ...(2)
De (1) y (2):
$x + y = m + n = 120°$
$\therefore x + y = 120°$

Clave B

22.



El $\triangle$ACD resulta notable de 53°/2.

El $\triangle$ABE resulta notable de 45°, entonces:

$x + \dfrac{53°}{2} = 45° \Rightarrow x = \dfrac{37°}{2}$

Clave D

Resolución de problemas

23. Sean las bases: a, b

$\dfrac{a-b}{2} = 8 \Rightarrow a - b = 16$

dato: $a + b = 30$

Resolviendo:

a = 23 m; b = 7 m

Clave C

24. Sean los lados paralelos (bases): a y b

$\Rightarrow a + b + 40 + 40 = 220$

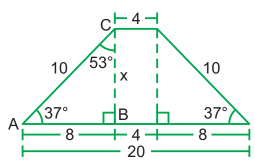
$a + b = 140$

La mediana es:

$\dfrac{a+b}{2} = \dfrac{140}{2} = 70$

Clave E

25. Del enunciado:



El $\triangle$ABC es notable (37° y 53°)

$\Rightarrow x = 6$

Clave C

26. Según el enunciado; si las bases son a, b (a menor que b):

$\dfrac{b+a}{2} + \dfrac{b-a}{2} = 40$

$\dfrac{2b}{2} = 40 \Rightarrow b = 40$

Clave D

Nivel 3 (página 43) Unidad 2

Comunicación matemática

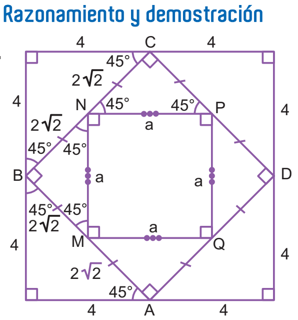
27.

28.

29.

Razonamiento y demostración

30.



Se deduce que el cuadrilátero MNPQ es un cuadrado.

En el $\triangle$NBM notable de 45°:

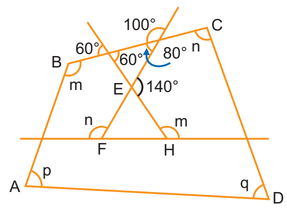
$a = \left(2\sqrt{2}\right)\sqrt{2} \Rightarrow a = 4$

Piden: perímetro MNPQ = 4a = 4(4)

$\therefore$ perímetro MNPQ = 16

Clave B

31.



En el $\triangle$EFH:

$n + m + 140° = 360°$

$n + m = 220°$ ...(1)

En el cuadrilátero ABCD:

$p + m + n + q = 360°$ ...(2)

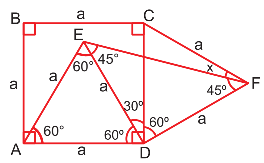
Reemplazando (1) en (2):

$p + 220° + q = 360°$

$\therefore p + q = 140°$

Clave D

32.



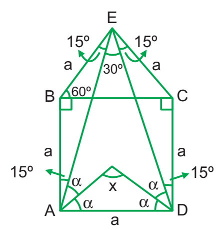
El $\triangle$EDF resulta notable de 45°.

Del gráfico:

$45° + x = 60° \Rightarrow x = 15°$

Clave E

33.



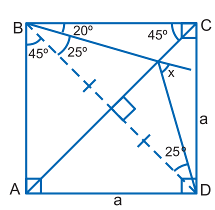
Por propiedad de los triángulos:

$x = 90° + \dfrac{30°}{2}$

$\therefore x = 105°$

Clave A

34.



Por ángulo exterior:

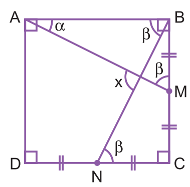
$x = 25° + 25°$

$\therefore x = 50°$

Clave E

Resolución de problemas

35. Según el enunciado:

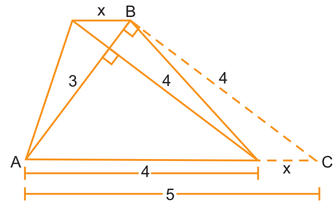


$\Rightarrow x = \alpha + \beta$

$\therefore x = 90°$

Clave B

36. Según el enunciado:

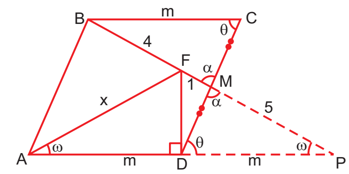


En el $\triangle$ABC: AC = 5

$\Rightarrow 4 + x = 5 \Rightarrow x = 1$ m

Clave D

37.



Prolongamos $\overline{BM}$ y $\overline{AD}$, entonces:

$\triangle BCM \cong \triangle PDM$ (Caso ALA)

$\Rightarrow BM = MP = 5 \quad \wedge \quad BC = DP$

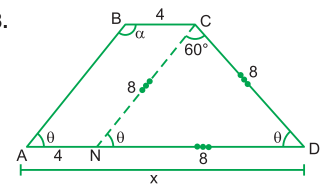
Luego se deduce que el $\triangle$AFP es isósceles.

$\Rightarrow AF = FP = 1 + 5$

$\therefore x = 6$

Clave B

38.



Por dato: ABCD es un trapecio isósceles

Del gráfico: $\alpha + \theta = 180°$

Pero: $\alpha = 90° + \dfrac{\theta}{2}$ (dato)

$\Rightarrow \left(90° + \dfrac{\theta}{2}\right) + \theta = 180°$

$\dfrac{3\theta}{2} = 90° \Rightarrow \theta = 60°$

Trazamos $\overline{CN} \,//\, \overline{AB}$, entonces: BC = AN = 4

Luego, como $\theta = 60°$

El $\triangle$NCD resulta equilátero.

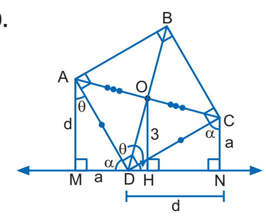
$\Rightarrow CN = CD = ND = 8$

Entonces: $x = 4 + 8$

$\therefore x = 12$

Clave A

39.



En el $\triangle$DMA: $\alpha + \theta = 90°$

Se tiene: $\triangle AMD \cong \triangle DNC$ (caso ALA)

En el trapecio ACNM:

$\dfrac{a+d}{2} = 3 \Rightarrow a + d = 6$

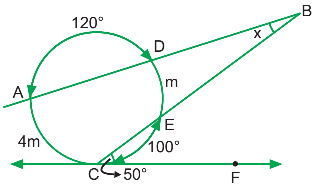
Piden: MN = a + d = 6

$\therefore$ MN = 6

Clave A

APLICAMOS LO APRENDIDO (página 45) Unidad 2

1. Piden: $m\angle ABC = x$



$4m + 120° + m + 100° = 360°$

$5m + 220° = 360°$

$5m = 140° \Rightarrow m = 28°$

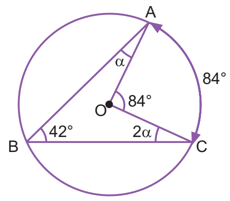
Por propiedad:

$x = \dfrac{4m - m}{2} = \dfrac{3m}{2}$

$\therefore x = \dfrac{3(28°)}{2} = 42°$

Clave D

2.



Por ángulo inscrito:

$m\angle ABC = \dfrac{84°}{2} = 42°$

Por ángulo central:

$m\angle AOC = 84°$

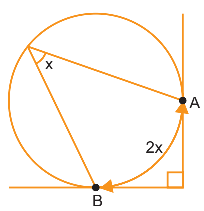
Por propiedad:

$\alpha + 42° + 2\alpha = 84°$

$3\alpha = 42° \Rightarrow \alpha = 14°$

Clave A

3.



Por ángulo inscrito:

$m\overset{\frown}{AB} = 2x$

Por ángulo exterior:

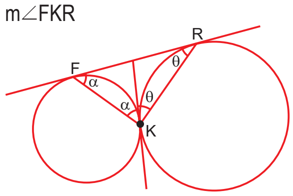
$m\overset{\frown}{AB} + 90° = 180°$

$2x + 90° = 180°$

$\therefore x = 45°$

Clave B

4. Piden: $m\angle FKR$



Piden: $x = \alpha + \theta$ ...(1)

En el triángulo FKR:

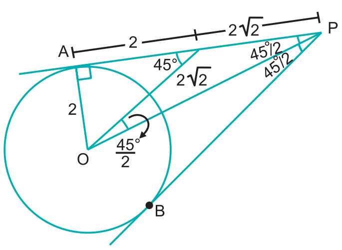
$\alpha + \alpha + \theta + \theta = 180°$

$\alpha + \theta = 90°$ ...(2)

Reemplazando (2) en (1): $\therefore x = 90°$

Clave B

5.

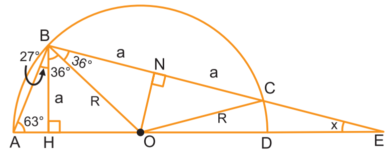


Luego:

$AP = 2 + 2\sqrt{2}$

Clave E

6.



El $\triangle AOB$ es isósceles

$\Rightarrow m\angle ABO = 63°$

Luego aplicando el teorema de la bisectriz:

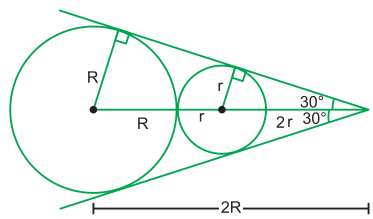
$m\angle NBO = 36°$

$\Rightarrow$ Del ⊿BHE:

$36° + 36° + x = 90° \Rightarrow x = 18°$

Clave D

7.



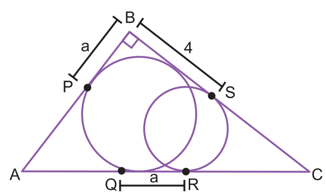
Del gráfico:

$2R = R + r + 2r$

$R = 3r \Rightarrow \dfrac{R}{r} = 3$

Clave C

8.



Del gráfico:

$AP = AQ$ y $RC = SC$

Aplicando el teorema de Poncelet:

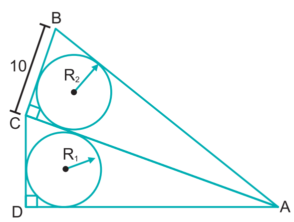
$(AP + a) + (4 + SC) = (AQ + a + RC) + 2r$

$4 = 2r$

$\therefore r = 2$

Clave B

9.



Dato: $AB = CD + DA$

Aplicando el teorema de Poncelet:

$CB + CA = AB + 2R_2$

$CD + DA = CA + 2R_1$ (+)

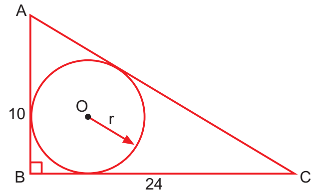
$CB = 2(R_1 + R_2)$

$10 = 2(R_1 + R_2)$

$\therefore R_1 + R_2 = 5$

Clave D

10.



Por el teorema de Pitágoras:

$(AC)^2 = 10^2 + 24^2 = 676 \Rightarrow AC = 26$

Por el teorema de Poncelet:

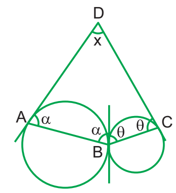
$AB + BC = AC + 2r$

$10 + 24 = 26 + 2r \Rightarrow 34 = 26 + 2r$

$\therefore r = 4$

Clave B

11. Piden: $m\angle ADC = x$



$m\angle ABC = 2m\angle ADC$

$\alpha + \theta = 2x$ ...(1)

En el polígono ABCD:

$2\alpha + 2\theta + x = 180°(4 - 2)$

$2(\alpha + \theta) + x = 360°$ ...(2)

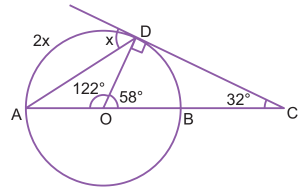
Reemplazando (1) en (2):

$2(2x) + x = 360°$

$5x = 360° \Rightarrow x = 72°$

Clave A

12. Piden: $m\angle ADE = x$

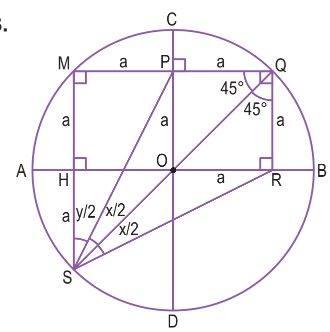


$\Rightarrow 2x = 122°$

$\therefore x = 61°$

Clave D

13.



Vemos que el $\triangle QSP \cong \triangle QSR$ (caso LAL) y según el dato:

PQRO es un cuadrado $(PQ = QR = RO = OP = a)$

$\Rightarrow m\angle PSQ = m\angle QSR = x/2$, luego prolongamos

$\overline{PQ}$ hasta que se interseca con la circunferencia en el punto M $\Rightarrow$ por propiedad: PQ = MP = a

Luego trazamos $\overline{MS}$; $\Rightarrow$ m$\angle$SMQ = 90° dado que $\overline{QS}$ es diámetro $\Rightarrow$ ▱MQRM es un rectángulo

$\therefore$ por propiedad: MH = HS = a

$\Rightarrow$ el ⊿PMS es notable de 53°/2;

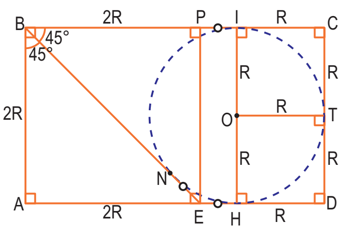
si $m\angle MSP = \frac{y}{2} = \frac{53}{2} \Rightarrow y = 53°$

Pero en el $\triangle$SMQ: $\frac{y}{2} + \frac{x}{2} = 45°$, reemplazando

$$\frac{53°}{2} + \frac{x}{2} = 45° \Rightarrow x = 37°$$

Clave D

14. Graficamos el rectángulo ABCD y trazamos los radios OI, OT y OH (donde OT = OI = OH = R).



Vemos que 2R = AB = CD y luego trazamos $\overline{PE}$ perpendicular a $\overline{BC}$, vemos que: BP = PE = 2R y por propiedad de la circunferencia:

BI = BN y NE = EH ... (I)

Del dato sabemos que: BN – NE = 16

BN = 16 + NE

Reemplazando en (I):

BI = 16 + EH; pero EH = PI $\Rightarrow$ BI = 16 + PI

$\Rightarrow$ BI – PI = 16 = 2R $\Rightarrow$ R = 8

Clave B

PRACTIQUEMOS

Nivel 1 (página 47) Unidad 2

Comunicación matemática

1.

I. Arco capaz	(III)
II. Ángulo exterior	(I)
III. Ángulo central	(IV)
IV. Ángulo semiinscrito	(II)

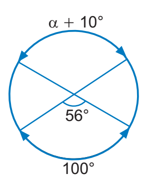
2. C. D. E. F.

3.

I. Circunferencia concéntricas	(III)
II. Circunferencias ortogonales	(I)
III. Circunferencias secantes	(II)

Razonamiento y demostración

4.



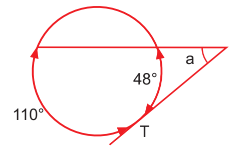
$$56° = \frac{100° + \alpha + 10°}{2}$$

$112° = 110° + \alpha$

$2° = \alpha$

Clave A

5.



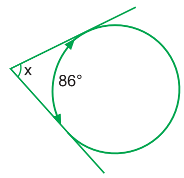
$a = \frac{110° - 48°}{2}$

$a = \frac{62°}{2}$

$a = 31°$

Clave C

6.

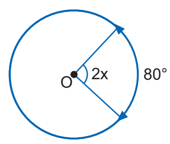


$x + 86° = 180°$

$x = 94°$

Clave A

7.

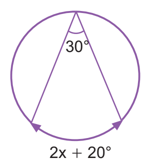


$2x = 80°$

$x = 40°$

Clave D

8.



$30° = \frac{2x + 20°}{2}$

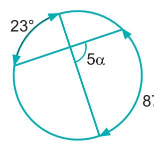
$60° = 2x + 20°$

$40° = 2x$

$20° = x$

Clave A

9.



$5\alpha = 55°$

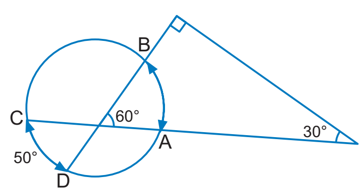
$\alpha = 11°$

Clave E

Resolución de problemas

10. Piden: $m\overset{\frown}{AB}$

Dato: $m\overset{\frown}{CD} = 50°$



Por propiedad:

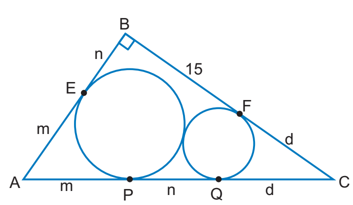
$$60° = \frac{m\overset{\frown}{CD} + m\overset{\frown}{BA}}{2}$$

$120° = 50° + m\overset{\frown}{BA}$

$\therefore m\overset{\frown}{BA} = 70°$

Clave E

11.



Sea r el radio de la circunferencia inscrita en el ⊿ABC.

Por el teorema de Poncelet:

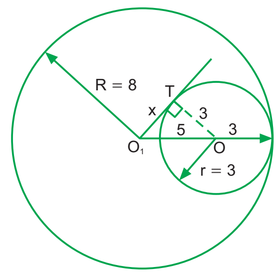
AB + BC = AC + 2r

(m + n) + (15 + d) = (m + n + d) + 2r

15 = 2r $\Rightarrow$ r = 7,5

Clave E

12.



En el ⊿O_1TO por el teorema de Pitágoras:

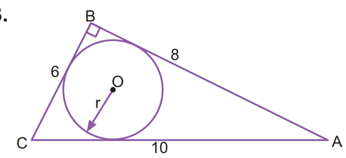
$5^2 = x^2 + 3^2$

$\Rightarrow x^2 = 16$

$\therefore x = 4$

Clave E

13.



Por el teorema de Pitágoras: BC = 6

Por el teorema de Poncelet:

AB + BC = AC + 2r

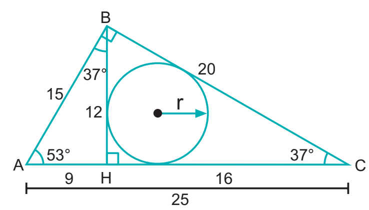
8 + 6 = 10 + 2r

4 = 2r

$\therefore$ r = 2

Clave B

14.



El ⊿ABC es notable de 37° y 53°.

En el ⊿BHC por el teorema de Poncelet:

BH + HC = BC + 2r

12 + 16 = 20 + 2r

8 = 2r

$\therefore$ r = 4

Clave B

Nivel 2 (página 48) Unidad 2

Comunicación matemática

15.

I. Las circunferencias de radios R_1 y R_2 son: circunferencias concéntricas.

II. Las circunferencias de radios R_3 y R_4 son: circunferencias tangentes interiores.

III. Las circunferencias de radios R_2 y R_4 son: circunferencias exteriores

IV. Las circunferencias de radios R_1 y R_3 son: circunferencias tangentes exteriores.

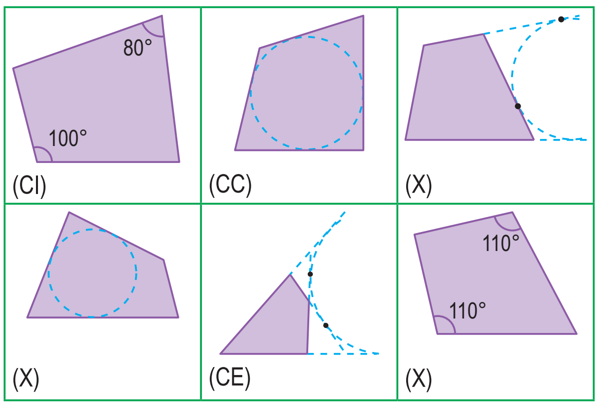
16.

I. Todos los triángulos son inscriptibles dentro de una circunferencia. (V)

II. Un cuadrilátero bicentro está inscrito y circunscrito a dos circunferencias concéntricas. (F)

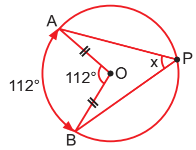
III. Todos los cuadriláteros son inscriptibles dentro de una circunferencia. (F)

17.



Razonamiento y demostración

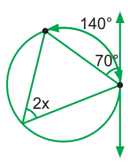
18. Piden: x



Por propiedad:
$112° = 2x$
$\therefore x = 56°$

Clave A

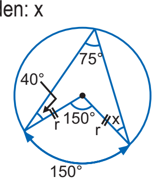
19. Piden: x



Por propiedad:
$4x = 140°$
$\therefore x = 35°$

Clave C

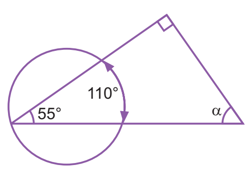
20. Piden: x



Por propiedad:
$40° + 75° + x = 150°$
$\therefore x = 35°$

Clave A

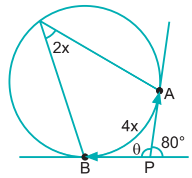
21. Piden: α



Del gráfico:
$55° + \alpha = 90°$
$\therefore \alpha = 35°$

Clave B

22.



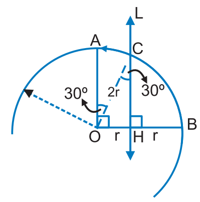
Del gráfico:
$\theta + 80° = 180° \Rightarrow \theta = 100°$

Por propiedad del ángulo exterior:
$4x + \theta = 180°$
$4x + 100° = 180°$
$4x = 80°$
$\therefore x = 20°$

Clave A

Resolución de problemas

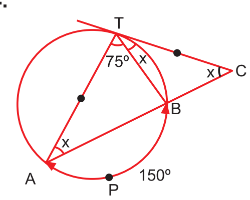
23.



Del gráfico:
$m\overset{\frown}{AC} = m\angle AOC$
$\therefore m\overset{\frown}{AC} = 30°$

Clave B

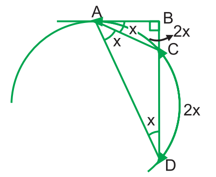
24.



En el ΔATC:
$3x + 75° = 180°$
$3x = 105°$
$\therefore x = 35°$

Clave C

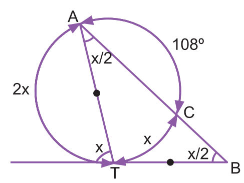
25.



En el ◺ABD:
$2x + x = 90°$
$3x = 90°$
$\therefore x = 30°$

Clave A

26.



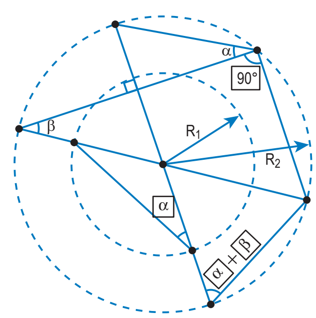
Del gráfico:
$2x + x + 108° = 360°$
$3x = 252°$
$\therefore x = m\overset{\frown}{TC} = 84°$

Clave D

Nivel 3 (página 49) Unidad 2

Comunicación matemática

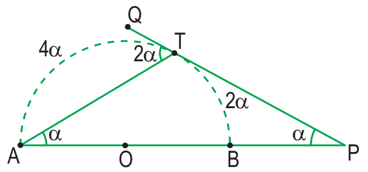
27.



28. Comenzamos asumiendo que el ΔATP es isósceles $\Rightarrow m\angle TPA = m\angle TAP = \alpha$

Por ángulo externo: $m\angle ATQ = 2\alpha$

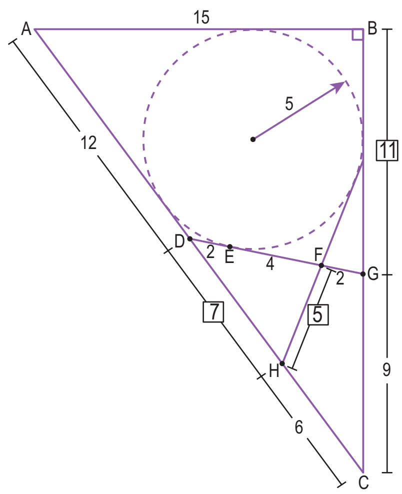
Por ángulo inscrito: $m\overset{\frown}{TB} = 2\alpha$



Por ángulo semiinscrito $m\overset{\frown}{AT} = 4\alpha$, pero $m\overset{\frown}{AB} = 180°$, pues $\overset{\frown}{AB}$ es una semicircunferencia $\Rightarrow$ $m\overset{\frown}{AT} + m\overset{\frown}{TB} = m\overset{\frown}{AB}$ reemplazando:

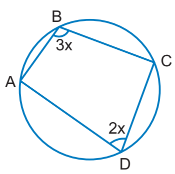
$4\alpha + 2\alpha = 180° \Rightarrow \alpha = 30°$

29.



Razonamiento y demostración

30.



Por cuadrilátero inscrito:

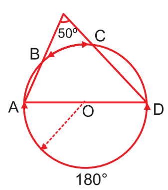
$2x + 3x = 180°$

$5x = 180°$

$\therefore x = 36°$

Clave C

31. Por ángulo exterior:



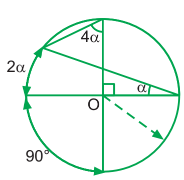
$$50° = \frac{180° - m\overset{\frown}{BC}}{2}$$

$$100° = 180° - m\overset{\frown}{BC}$$

$$\therefore m\overset{\frown}{BC} = 80°$$

Clave E

32.



Por ángulo inscrito:

$$\frac{90° + 2\alpha}{2} = 4\alpha$$

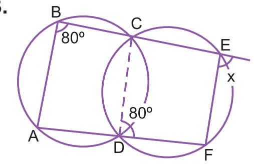
$$90° + 2\alpha = 8\alpha$$

$$90° = 6\alpha$$

$$\Rightarrow \alpha = 15°$$

Clave B

33.



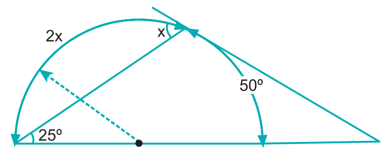
Por cuadrilátero inscrito:

$x = m\angle CDF$

$\therefore x = 80°$

Clave C

34. Del gráfico:



$2x + 50° = 180°$

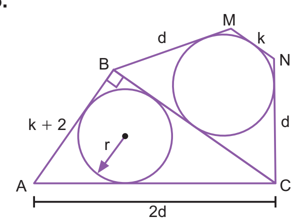
$2x = 130°$

$\therefore x = 65°$

Clave C

Resolución de problemas

35.



En el cuadrilátero BMNC por el teorema de Pitot:

$k + BC = d + d$

$\Rightarrow BC = 2d - k$

En el $\triangle$ABC por el teorema de Poncelet:

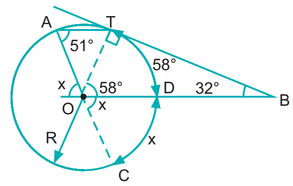
$AB + BC = AC + 2r$

$(k + 2) + (2d - k) = 2d + 2r$

$2 = 2r \Rightarrow r = 1$

Clave C

36. Por ángulo inscrito:



$m\overset{\frown}{TC} = 2(m\angle TAC)$

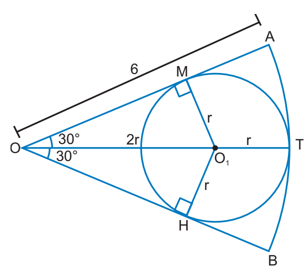
$58° + x = 2(51°)$

$58° + x = 102°$

$\therefore x = 44°$

Clave A

37.



El $\triangle OO_1H$ es notable de 30° y 60°.

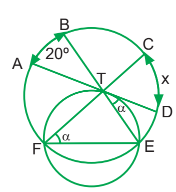
$\Rightarrow OO_1 = 2r$

Del gráfico: $3r = 6$

$\therefore r = 2$

Clave A

38. Por ángulo interior:



$$\frac{m\overset{\frown}{AB} + m\overset{\frown}{DE}}{2} = \alpha$$

$$20° + m\overset{\frown}{DE} = 2\alpha$$

$$\Rightarrow m\overset{\frown}{DE} = 2\alpha - 20°$$

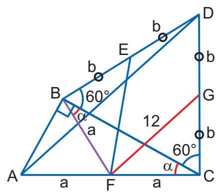
Del gráfico: $m\overset{\frown}{CE} = 2\alpha$

$x + 2\alpha - 20° = 2\alpha \Rightarrow x = m\overset{\frown}{CD} = 20°$

Clave A

MARATÓN MATEMÁTICA (página 51)

1.



Si $AD = 24$

En el $\triangle ADC$, se traza $\overline{FG}$ paralelo a $\overline{AD}$

$\Rightarrow FG = \dfrac{AD}{2} \Rightarrow FG = 12$

Se traza la mediana $\overline{BF}$

$BF = FC = AF$

$\Rightarrow m\angle FBC = m\angle FCB = \alpha$

Del $\triangle FBE$ y $\triangle FCG$ (caso LAL).

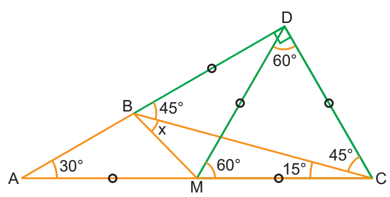
$\Rightarrow \triangle FBE \cong \triangle FCG$

$\therefore\ EF = FG = 12$

$EF = 12$

Clave D

2.



Se traza la altura DC relativa a la prolongación de $\overline{AB}$.

$\Rightarrow m\angle CBD = m\angle BAC + m\angle ACB = 30° + 15° = 45°$.

El ⊿ ADC, es un triángulo rectángulo notable.

$\Rightarrow\ m\angle ACD = 60° = m\angle ACB + m\angle BCD$

$\Rightarrow\ m\angle BCD = 45°$

$\therefore$ ⊿BDC es isósceles

Por propiedad de triángulos rectos:

$AM = MD = MC$

$\Rightarrow$ En el $\triangle MDC$, es un triángulo, isósceles de 60°, entonces es un $\triangle$equilátero.

$\therefore\ AM = MC = MD = BD = DC$

En el $\triangle BDM$, es un $\triangle$isósceles:

$\Rightarrow m\angle DBM = m\angle BMD = 75°$

$45 + x = 75° \Rightarrow x = 30°$

Clave B

3. Recordemos: $D_k = kn - \dfrac{(k+1)k}{2}$, $k = n - 5$

D_k: número de diagonales medias desde k lados consecutivos

$\Rightarrow 8n - 10 = (n-5)(n) - \dfrac{(n-5)(n-5+1)}{2}$

$n^2 = 16\,n$

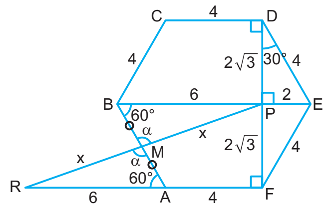
$n = 16$

Nos piden: $D_M = \dfrac{n(n-1)}{2}$

$D_M = \dfrac{16(15)}{2} = 120$

Clave B

4.



Se prolonga $\overline{AF}$ y $\overline{PM}$ y se interseca en R.

$\overline{BE} \,//\, \overline{AF} \Rightarrow m\angle RAB = m\angle PBA$

Por congruencia (caso ALA):

En los $\triangle RMA$ y $\triangle BMP$

$RM = MP = x$ y $RA = BP = 6$

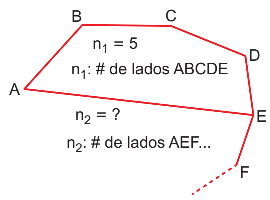
En el $\triangle RFP$:

$(2x)^2 = 10^2 + (2\sqrt{3})^2$

$4x^2 = 112 \Rightarrow x = 2\sqrt{7}$

Clave D

5.



n: # de lados del polígono ABCDEF...

Por dato:

$\dfrac{n_2(n_2-3)}{2} - \dfrac{n_1(n_1-3)}{2} = 9 \ \wedge\ n_1 = 5$

$\dfrac{n_2(n_2-3)}{2} - \dfrac{5(5-3)}{2} = 9$

$\Rightarrow\ n_2^2 - 3n_2 = 28 \Rightarrow n_2^2 - 3n_2 - 28 = 0$

$n_2 \quad -7$

$n_2 \quad 4$

$n_2 = 7 \wedge n_2 = -4$ (no cumple)

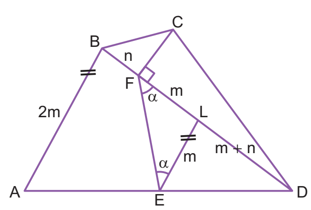
$\therefore\ n = (n_1 - 1) + (n_2 - 1) = 5 + 7 - 2 = 10$

$\Sigma m\angle i_{ABCDEF...} = 180°(n-2)$

$= 1440°$

Clave B

6.



Si: $AB = 2m$ y $BF = n \Rightarrow BD = 2m + 2n$

Del punto E se traza $\overline{EL} \,//\, \overline{AB}$.

$\Rightarrow$ en el $\triangle ABD$:

$BL = LD = \dfrac{2m+2n}{2}$ y $LE = \dfrac{2m}{2}$

Si: $LD = m + n \Rightarrow FL = m \Rightarrow m\angle LFE = m\angle FEL = \alpha$

$m\angle ABD < 40°$ y $\overline{AB} \,//\, \overline{EL} \Rightarrow m\angle ABD = m\angle ELD$

En el $\triangle FLE$: $2\alpha < 40 \Rightarrow \alpha < 20°$

Piden el mayor valor entero de $m\angle CFE$:

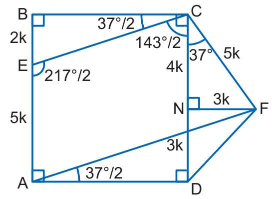
$m\angle CFE = 90 + \alpha$ y $\alpha < 20° \Rightarrow 90 + \alpha < 110°$

$\Rightarrow m\angle CFE = 90° + \alpha < 110°$

$\therefore\ m\angle CFE = 109°$

Clave D

7.



Del trapecio isósceles AECF:

$m\angle AEC = m\angle ECF$, por ángulos externos.

$m\angle AEC = \dfrac{217°}{2} \Rightarrow m\angle DCF = 37°$

Si $AE = 5k$

$AE = CF = 5k$, $EB = 2k$ y $AB = CD$

Del $\triangle CNF$:

$CN = 4k \ \wedge\ NF = 3k$

$\Rightarrow AB = CN + ND \Rightarrow 7k = 4k + ND \Rightarrow ND = 3k$

Del ⊿DNF:

$m\angle DFN = 45°$

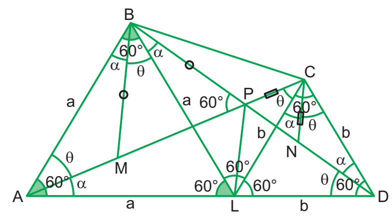
$\therefore$ Piden $m\angle DFC$

$m\angle DFE = m\angle NFC + m\angle NFD$

$m\angle DFE = 53° + 45° = 98°$

Clave A

8.



Se ubica L en $\overline{AD}$ tal que $AL = a$ y $LD = b$

$\Rightarrow$ Si $m\angle BAC = m\angle ADC = 60°$, los $\triangle ABL$ y $\triangle LCD$ son equiláteros y $m\angle BLC = 60°$.

Por congruencia (caso LAL)

$\triangle ALC \cong \triangle BLD$

$\Rightarrow m\angle CAL = m\angle LBD = \alpha$ y

$m\angle ACL = m\angle BDL = \theta$

Por ángulo exterior en el $\triangle ACL$:

$\alpha + \theta = 60°$

$\Rightarrow\ m\angle BAP = \theta$ y $m\angle PDC = \alpha$

Por dato $MB = BP$ y $PC = CN$

$\Rightarrow$ los triángulos MBP y PCN son equiláteros.

$\therefore\ m\angle MBL = \theta$ y $m\angle NCL = \alpha$

Entonces $m\angle ABM = \alpha$ y $m\angle NCD = \theta$

Por congruencia (caso LAL)

$\triangle ABM \cong \triangle LBP$

$\Rightarrow AM = LP$ ... (I)

Por congruencia (caso LAL)

$\triangle LPC \cong \triangle NCD$

$\Rightarrow LP = ND$ ... (II)

De (I) y (II):

$AM = LP = ND \Rightarrow AM = ND$

$\therefore\ \dfrac{AM}{ND} = 1$

Clave B

Unidad 3 PROPORCIONALIDAD

APLICAMOS LO APRENDIDO (página 53) Unidad 3

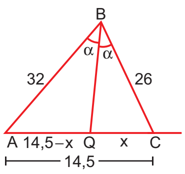
1. Por el teorema de Tales:

$$\frac{x+2}{x+8} = \frac{x}{x+3}$$

$$(x+2)(x+3) = x(x+8)$$
$$x^2 + 3x + 2x + 6 = x^2 + 8x$$
$$6 = 3x$$
$$\therefore\ x = 2$$

Clave C

2.

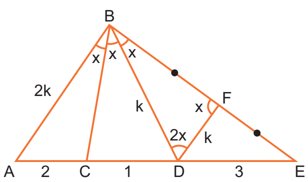


Por el teorema de la bisectriz interior:

$$\frac{32}{14,5-x} = \frac{26}{x} \Rightarrow 32x = 26(14,5-x)$$
$$32x = 377 - 26x$$
$$\therefore\ x = 6,5$$

Clave C

3. Por el teorema de la bisectriz interior:



$$\frac{AB}{2} = \frac{BD}{1}$$

$\Rightarrow AB = 2BD$

Si: $BD = k \Rightarrow AB = 2k$

Trazamos $\overline{DF}$ paralela a $\overline{AB}$, entonces:

$$\frac{EF}{FB} = \frac{3}{3} \Rightarrow EF = FB$$

Por el teorema de los puntos medios:

$$DF = \frac{AB}{2} \Rightarrow DF = k$$

Por lo tanto, el $\triangle BDF$ es isósceles:

$4x = 180°$

$\therefore\ x = 45°$

Clave C

4. Por el teorema de Tales:

$$\frac{x}{3x+2} = \frac{2}{2y+1} \quad ...(1)$$

$$\frac{2y+1}{3x+2} = \frac{2}{x} \quad ...(2)$$

$$\frac{3x+2}{6} = \frac{2y+1}{y} \Rightarrow \frac{2y+1}{3x+2} = \frac{y}{6} \quad ...(3)$$

Igualando (2) y (3):

$$\frac{2}{x} = \frac{y}{6} \Rightarrow xy = 12$$

Reemplazando en (1):

$2xy + x = 6x + 4 \Rightarrow 24 + x = 6x + 4$

$\therefore\ x = 4$

Clave C

5. Según el enunciado:

$$\frac{16k-15}{k+4} = \frac{10k+14}{6k}$$

Resolviendo: $k = 2$

$\therefore\ BC = k + 4 \Rightarrow 2 + 4 = 6$

Clave D

6. Del gráfico:

$$\frac{x+3}{8} = \frac{9}{x-3} \Rightarrow x^2 - 9 = 72$$
$$x^2 = 81 \Rightarrow x = 9$$

También:

$$\frac{8}{x+1} = \frac{y}{y+1}$$
$$\frac{8}{10} = \frac{y}{y+1}$$
$$8y + 8 = 10y$$
$$8 = 2y$$
$$\Rightarrow y = 4$$

Luego:

$xy = (9)(4) = 36$

Clave A

7. De la figura:

$$\frac{CA}{AE} = \frac{x+1}{2x} \quad ...(1)$$

$$\frac{CA}{AE} = \frac{9}{3x} \quad ...(2)$$

De (1) y (2):

$$\frac{x+1}{2x} = \frac{9}{3x}$$

Resolviendo:

$3x + 3 = 18$

$3x = 15 \Rightarrow x = 5$

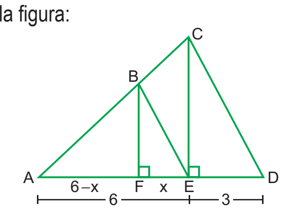
Clave E

8. Por propiedad (cuaterna armónica):

$$\frac{4}{3} = \frac{7+m}{m} \Rightarrow 4m = 21 + 3m$$
$$\therefore\ m = 21$$

Clave C

9. De la figura:



$$\frac{AB}{BC} = \frac{AE}{ED} \Rightarrow \frac{AB}{BC} = \frac{6}{3} \quad ...(1)$$

$$\frac{AB}{BC} = \frac{6-x}{x} \quad ...(2)$$

De (1) y (2): $\dfrac{6}{3} = \dfrac{6-x}{x}$

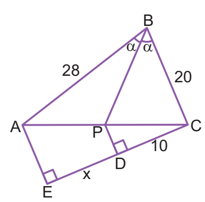
Resolviendo:

$6x = 18 - 3x$

$9x = 18 \Rightarrow x = 2$

Clave C

10.



Por el teorema de la bisectriz interior:

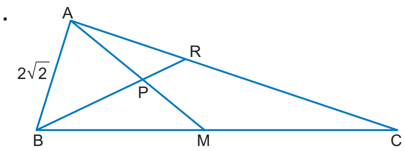
$$\frac{AP}{PC} = \frac{28}{20} \Rightarrow AP = 7k \ \wedge\ PC = 5k$$

Como $\overline{AE} \,//\, \overline{PD}$, por propiedad:

$$\frac{PC}{AP} = \frac{10}{x} \Rightarrow \frac{5k}{7k} = \frac{10}{x}$$
$$x = \frac{70}{5}$$
$$\therefore x = 14$$

Clave C

11.



Si $\overline{AM}$ es mediana: $\overline{BM} \cong \overline{MC}$

Luego:

$AM = 6AP$, si $AP = a \Rightarrow PM = 5a$

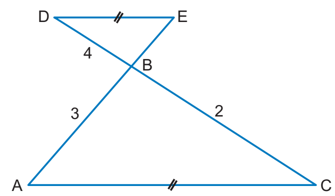
Por teorema de la bisectriz interior:

$$\frac{2\sqrt{2}}{a} = \frac{BM}{5a} \Rightarrow BM = 10\sqrt{2}$$

$\therefore\ BC = 2BM = 20\sqrt{2}$

Clave A

12. Del enunciado:

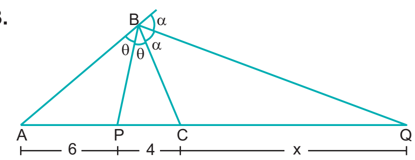


Por teorema de Thales:

$$\frac{BE}{AB} = \frac{DB}{BC} \Rightarrow \frac{BE}{3} = \frac{4}{2} \qquad \therefore\ BE = 6$$

Clave B

13.



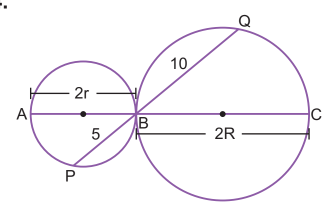
De la figura; A; P; C y Q son conjugados armonicos

$$\frac{AP}{CP} = \frac{AQ}{CQ} \Rightarrow \frac{6}{4} = \frac{10+x}{x}$$

$\therefore\ x = 20$

Clave E

14.



Por circunferencia tangentes exteriores:

$$\frac{BC}{AB} = \frac{BQ}{PB} \Rightarrow \frac{2R}{2r} = \frac{10}{5} \qquad \therefore\ \frac{r}{R} = \frac{1}{2}$$

Clave C

PRACTIQUEMOS

Nivel 1 (página 55) Unidad 3

Comunicación matemática

1.

2.

3.

Razonamiento y demostración

4. Del gráfico:

$$\frac{3}{5} = \frac{12}{x+4} \Rightarrow 3x + 12 = 60$$

$$3x = 48 \Rightarrow x = 16$$

Clave B

5. De la figura:

$$\frac{3}{3{,}5 - x} = \frac{4}{x} \Rightarrow 3x = 14 - 4x$$

$$7x = 14 \Rightarrow x = 2$$

Clave C

6. Del gráfico:

$$\frac{x}{9} = \frac{3}{3x} \Rightarrow x^2 = 9 \qquad \therefore\ x = 3$$

Clave C

7. Propiedad de la bisectriz:

$$\frac{28}{12+x} = \frac{20}{x} \Rightarrow \frac{7}{12+x} = \frac{5}{x} \Rightarrow 7x = 60 + 5x$$

$$2x = 60$$

$$\therefore\ x = 30$$

Clave E

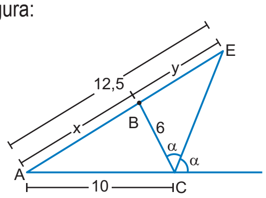
8. De la figura:

$$\frac{10}{15} = \frac{20}{x} \Rightarrow 10x = 300 \qquad \therefore\ x = 30$$

Clave D

Resolución de problemas

9. De la figura:

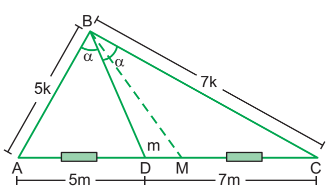


$$\frac{12{,}5}{y} = \frac{10}{6} \Rightarrow \frac{2{,}5}{y} = \frac{2}{6}$$

$$\frac{2{,}5}{y} = \frac{1}{3} \Rightarrow y = 7{,}5 \qquad \therefore\ x = 5$$

Clave B

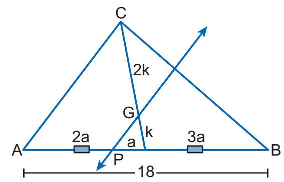
10.



$$\frac{DM}{AC} = \frac{m}{12m} \Rightarrow \frac{DM}{AC} = \frac{1}{12}$$

Clave D

11.

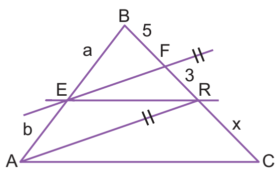


$6a = 18 \Rightarrow a = 3$

Luego: $PB = 4a = 4(3) = 12$

Clave B

12.



$$\frac{5}{3} = \frac{a}{b} = \frac{8}{x} \Rightarrow \frac{5}{3} = \frac{8}{x} \qquad \therefore\ x = 4{,}8$$

Clave B

Nivel 2 (página 56) Unidad 3

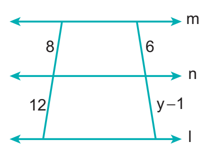
Comunicación matemática

13.

14.

Razonamiento y demostración

15.

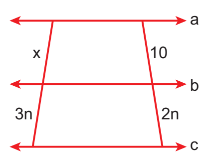


$\overleftrightarrow{m} \parallel \overleftrightarrow{n} \parallel \overleftrightarrow{t}$

$$\frac{8}{12} = \frac{6}{y-1} \Rightarrow y - 1 = 9 \Rightarrow y = 10$$

Clave B

16.

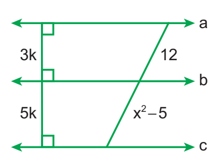


$$\frac{x}{3n} = \frac{10}{2n}$$

$$\therefore\ x = 15$$

Clave C

17.

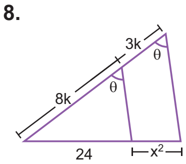


$$\frac{3k}{5k} = \frac{12}{x^2 - 5} \Rightarrow x^2 - 5 = 20$$

$$\Rightarrow x^2 = 25 \Rightarrow x = 5$$

Clave C

18.

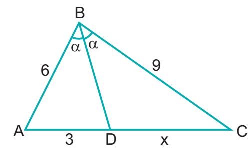


$$\frac{8k}{3k} = \frac{24}{x^2}$$

$$x^2 = 9 \Rightarrow x = 3$$

Clave C

19.

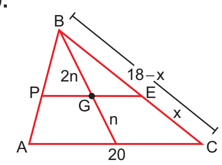


$$\frac{6}{3} = \frac{9}{x} \Rightarrow x = 4{,}5$$

Clave B

Resolución de problemas

20.



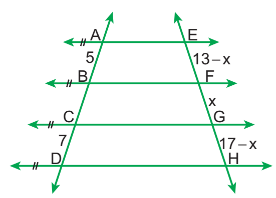
$$\frac{2n}{n} = \frac{18 - x}{x}$$

$$2x = 18 - x$$

$$\therefore\ x = 6$$

Clave B

21.



Piden: $FG = x$

Por el teorema de Tales:

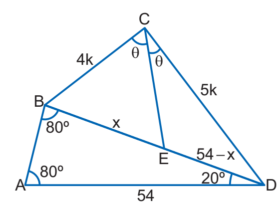
$$\frac{5}{7} = \frac{13 - x}{17 - x} \Rightarrow 85 - 5x = 91 - 7x$$

$$2x = 6$$

$$\therefore x = 3$$

Clave C

22.



Por dato: $5(BC) = 4(CD)$

$\Rightarrow BC = 4k$ y $CD = 5k$

El $\triangle BDA$ es isósceles, entonces:

$AD = BD = 54$

En el $\triangle BCD$ por el teorema de la bisectriz interior:

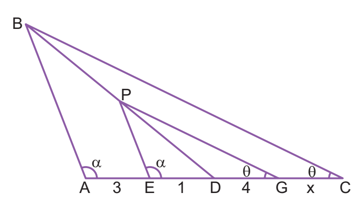
$$\frac{4k}{5k} = \frac{x}{54 - x} \Rightarrow 216 - 4x = 5x$$

$$216 = 9x$$

$$\therefore x = 24$$

Clave B

23.



Del gráfico: $\overline{AB} \parallel \overline{PE}$ y $\overline{PG} \parallel \overline{BC}$

Por el teorema de Tales:

$$\frac{BP}{PD} = \frac{3}{1} \wedge \frac{BP}{PD} = \frac{x}{4} \Rightarrow \frac{3}{1} = \frac{x}{4}$$

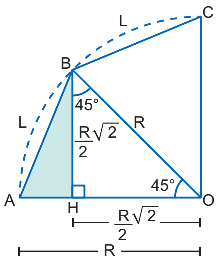
$$\therefore x = 12$$

Clave B

Nivel 3 (página 57) Unidad 3

Comunicación matemática

24.



Hallamos la razón entre el radio de una circunferencia y el lado del octágono inscrito en ella.

Ángulo central: $m\angle AOB = \frac{360°}{8} = 45°$

$\Rightarrow \triangle BHO$ es notable de 45°

$\therefore\ BH = HO = \frac{R}{2}\sqrt{2}$

$\Rightarrow \triangle AHB$: Teorema de Pitágoras

$L^2 = \left(\frac{R}{2}\sqrt{2}\right)^2 + \left(R - \frac{R}{2}\sqrt{2}\right)^2$

$L^2 = R^2(2 - \sqrt{2})$

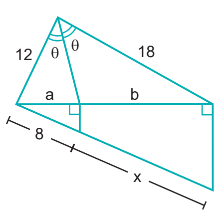
$L = R(\sqrt{2 - \sqrt{2}}) \Rightarrow \frac{R}{L} = \frac{1}{\sqrt{2 - \sqrt{2}}}$

$\Rightarrow \frac{R}{L} = 1.30656 = \frac{4,5}{h} \Rightarrow h = 3,444$

25.

Razonamiento y demostración

26.

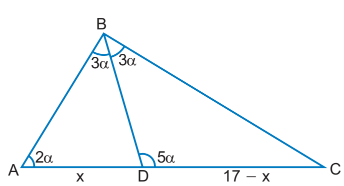


$\frac{12}{a} = \frac{18}{b} \quad \wedge \quad \frac{8}{x} = \frac{a}{b} \Rightarrow \frac{12}{18} = \frac{8}{x}$

$\therefore x = 12$

Clave C

27.



Por dato: $\frac{AB}{BC} = \frac{6}{11}$

Por el teorema de la bisectriz interior:

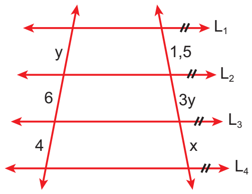
$\frac{AB}{BC} = \frac{x}{17 - x} \Rightarrow \frac{6}{11} = \frac{x}{17 - x}$

$102 - 6x = 11x$

$102 = 17x \Rightarrow x = 6$

Clave A

28.



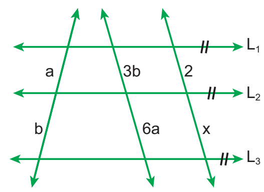
Por el teorema de Tales:

- $\frac{y}{6} = \frac{1,5}{3y} \Rightarrow y^2 = 3$
  $y = \sqrt{3}$
- $\frac{6}{4} = \frac{3y}{x} \Rightarrow x = 2y = 2(\sqrt{3})$

$\therefore x = 2\sqrt{3}$

Clave B

29.



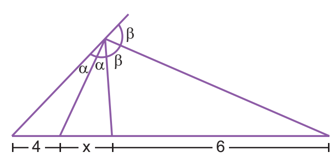
Por el teorema de Tales:

- $\frac{a}{b} = \frac{3b}{6a} \Rightarrow \frac{a}{b} = \frac{1}{\sqrt{2}}$
- $\frac{a}{b} = \frac{2}{x} \Rightarrow \frac{1}{\sqrt{2}} = \frac{2}{x}$

$\therefore x = 2\sqrt{2}$

Clave C

30.



Por cuaterna armónica:

$\frac{4}{x} = \frac{10 + x}{6} \Rightarrow 24 = x(x + 10)$

$2\,(12) = x(x + 10)$

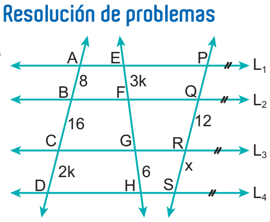
$2(2 + 10) = x(x + 10)$

$\therefore x = 2$

Clave B

Resolución de problemas

31.



Por el teorema de Tales:

$\frac{AB}{BC} = \frac{EF}{FG} \Rightarrow \frac{8}{16} = \frac{3k}{FG} \Rightarrow FG = 6k$

$\frac{CD}{BC} = \frac{GH}{FG} \Rightarrow \frac{2k}{16} = \frac{6}{6k} \Rightarrow k^2 = 8$

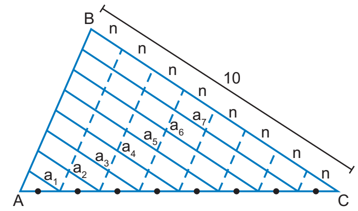
$k = 2\sqrt{2}$

$\frac{FG}{GH} = \frac{QR}{RS} \Rightarrow \frac{6k}{6} = \frac{12}{x} \Rightarrow x = \frac{12}{k} = \frac{12}{2\sqrt{2}}$

$\therefore x = 3\sqrt{2}$

Clave B

32.



Se trazan paralelas a $\overline{AB}$ con lo cual se forman paralelogramos, entonces:

$a_1 = n$

$a_2 = 2n$

$a_3 = 3n$

$\vdots$

$a_7 = 7n$

Del gráfico: $8n = 10 \Rightarrow n = \frac{5}{4}$

Piden:

$a_1 + a_2 + a_3 + ... + a_7 = n(1 + 2 + 3 + ... + 7)$

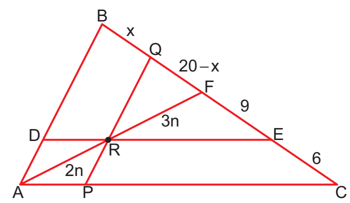
$= n\left(\frac{7 \cdot 8}{2}\right) = 28n$

$= 28\left(\frac{5}{4}\right)$

$\therefore a_1 + a_2 + a_3 + ... + a_7 = 35$

Clave E

33.



Por dato: $\overline{DE} \,//\, \overline{AC} \wedge \overline{PQ} \,//\, \overline{AB}$

Por el teorema de Tales:

$FR = 3n \wedge AR = 2n$

Además:

$\frac{FQ}{QB} = \frac{FR}{RA} \Rightarrow \frac{20 - x}{x} = \frac{3n}{2n}$

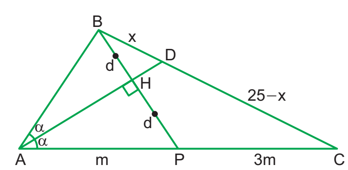
$40 - 2x = 3x$

$40 = 5x$

$\therefore x = 8$

Clave B

34.



Por el teorema de Menelao:

$CD \,.\, BH \,.\, PA = BD \,.\, HP \,.\, CA$

$(25 - x) \,.\, d \,.\, m = x \,.\, d \,.\, 4m$

$25 - x = 4x$

$25 = 5x$

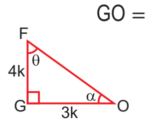
$\therefore x = 5$

Clave B

APLICAMOS LO APRENDIDO (página 59) Unidad 3

1. Por ser G baricentro.
VN = NA y FG = 2GN ...(1)
Por el teorema de la bisectriz interior:
VF = FA $\Rightarrow$ ΔVFA es isósceles.
$2\alpha + 2\theta = 180° \Rightarrow \alpha + \theta = 90°$
Del dato: 2GO = 3GN y de (1):

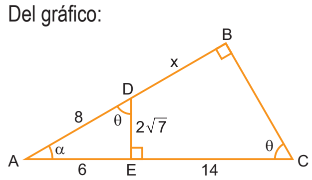
$$GO = \frac{3FG}{4} \Rightarrow FG = 4k \wedge GO = 3k$$



$\therefore \theta = 37°$

Clave C

2. Del gráfico:



ΔAED ~ ΔABC

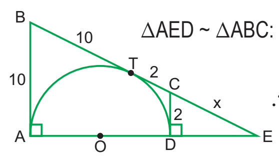
$$\frac{x+8}{6} = \frac{6+14}{8} \Rightarrow x+8 = \frac{120}{8}$$

$x + 8 = 15$

$\therefore x = 7$

Clave C

3.



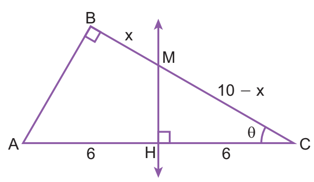
ΔAED ~ ΔABC: $\dfrac{x}{2} = \dfrac{12+x}{10}$

$5x = 12 + x$

$\therefore x = 3$

Clave B

4.



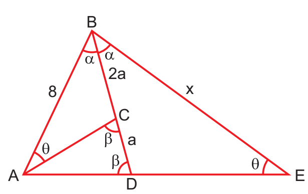
$\triangle$CHM ~ $\triangle$CBA: $\dfrac{6}{10-x} = \dfrac{10}{12}$

$10 - x = 7{,}2$

$\therefore x = 2{,}8$

Clave C

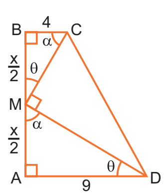
5.



ΔABC ~ ΔEBD: $\dfrac{2a}{8} = \dfrac{3a}{x} \Rightarrow x = 12$

Clave D

6.



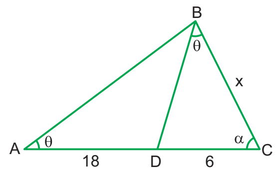
$\triangle$MBC ~ $\triangle$DAM

$$\frac{\frac{x}{2}}{4} = \frac{9}{\frac{x}{2}}$$

$$\left(\frac{x}{2}\right)^2 = 36 \Rightarrow x = 12$$

Clave D

7.

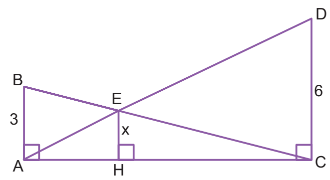


Del gráfico: ΔACB ~ ΔBCD:

$\dfrac{24}{x} = \dfrac{x}{6} \Rightarrow x^2 = 144$ $\therefore x = 12$

Clave E

8.

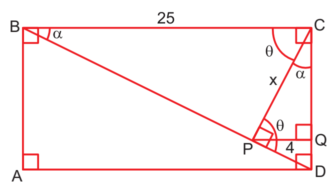


Por propiedad:

$x = \dfrac{(3)(6)}{3+6} = \dfrac{18}{9}$ $\therefore x = 2$

Clave A

9.



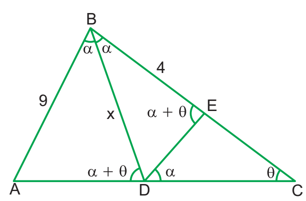
Del gráfico: ΔBCP ~ ΔCPQ

$\dfrac{x}{4} = \dfrac{25}{x} \Rightarrow x^2 = 100$

$\therefore x = 10$

Clave B

10.

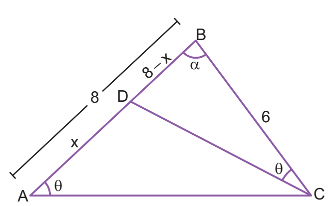


Del gráfico: ΔABD ~ΔDBE:

$\dfrac{9}{x} = \dfrac{x}{4} \Rightarrow x^2 = 36$ $\therefore x = 6$

Clave C

11.



Del gráfico: ΔABC ~ ΔCBD:

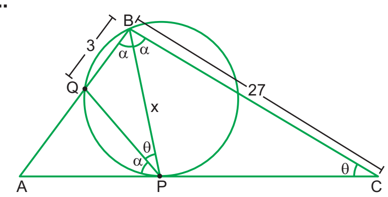
$\dfrac{6}{8-x} = \dfrac{8}{6} \Rightarrow 36 = 64 - 8x$

$8x = 28$

$\therefore x = 3{,}5$

Clave C

12.



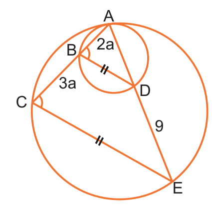
Se traza $\overline{QP}$, ΔBQP ~ ΔBPC:

$$\frac{BP}{BC} = \frac{BQ}{BP} \Rightarrow \frac{x}{27} = \frac{3}{x}$$

$\therefore x = 9$

Clave C

13.

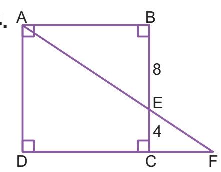


$\overline{BD}$ // $\overline{CE}$; ΔABD ~ ΔACE:

$$\frac{AE}{AD} = \frac{AC}{AB} \Rightarrow \frac{AE}{AE-9} = \frac{5a}{2a} \quad \therefore AE = 15$$

Clave E

14.



En el $\triangle$ABE, AB = 12

$AE^2 = 8^2 + 12^2$

$AE = 4\sqrt{13}$

$\triangle$ECF ~ $\triangle$EBA:

$$\frac{EF}{AE} = \frac{EC}{BE} \Rightarrow \frac{EF}{4\sqrt{13}} = \frac{4}{8}$$

$\therefore EF = 2\sqrt{13}$

Clave A

Nivel 1 (página 61) Unidad 3

Comunicación matemática

1.

2.

3.

Razonamiento y demostración

4. De los triángulos semejantes:

$\dfrac{2}{x} = \dfrac{x}{8} \Rightarrow x^2 = 16$

$\therefore x = 4$

Clave D

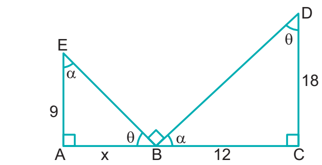
5. Son triángulos semejantes:

$\dfrac{x}{4} = \dfrac{9}{x} \Rightarrow x^2 = 36$

$\therefore x = 6$

Clave C

6.

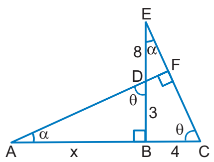


$\triangle EAB \sim \triangle BCD$:

$\frac{x}{18} = \frac{9}{12} \Rightarrow x = 13{,}5$

Clave B

7.



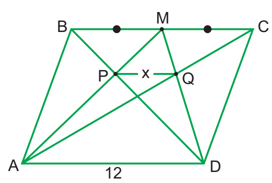
$\triangle ABD \sim \triangle EBC$:

$\frac{3}{4} = \frac{x}{11} \Rightarrow x = \frac{33}{4}$

Clave B

Resolución de problemas

8. Según el enunciado:



En el $\triangle ABC$: P es baricentro

$\Rightarrow PM = a \ \wedge \ AP = 2a$

En el $\triangle BDC$: Q es baricentro.

$\Rightarrow MQ = b \ \wedge \ QD = 2b$

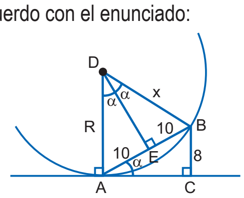
Luego: $\overline{PQ} \parallel \overline{AD}$

$\triangle AMD \sim \triangle PMQ$

$\frac{x}{12} = \frac{a}{3a} \Rightarrow x = \frac{12a}{3a} = 4$

Clave A

9. De acuerdo con el enunciado:



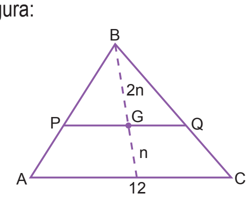
Se observa:

$\triangle AED \sim \triangle BCA$:

$\frac{10}{8} = \frac{R}{20} \Rightarrow R = 25$ cm

Clave D

10. De la figura:



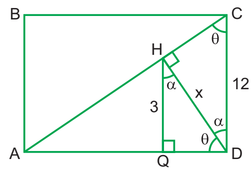
$\triangle ABC \sim \triangle PBQ$:

$\frac{12}{PQ} = \frac{3n}{2n} \Rightarrow PQ = \frac{12 \times 2n}{3n}$

$PQ = 8$

Clave E

11. Según el enunciado:



$\triangle HQD \sim \triangle DHC$

$\frac{3}{x} = \frac{x}{12} \Rightarrow 36 = x^2 \qquad \therefore x = 6$

Clave C

Nivel 2 (página 62) Unidad 3

Comunicación matemática

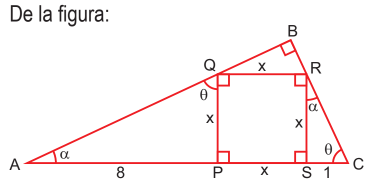
12.

13.

14.

Razonamiento y demostración

15. De la figura:

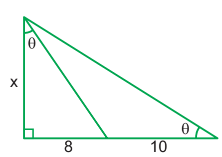


$\triangle APQ \sim \triangle RSC$:

$\frac{x}{1} = \frac{8}{x} \Rightarrow x^2 = 8 \qquad \therefore x = 2\sqrt{2}$

Clave C

16.



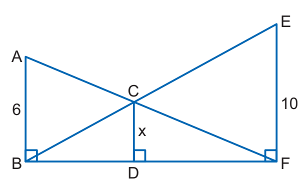
Por propiedad: $x^2 = (10 + 8)8$

$x^2 = 144$

$x = 12$

Clave C

17.

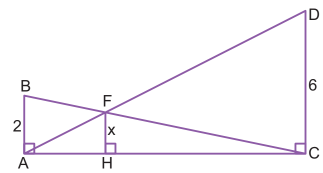


Por propiedad: $\frac{1}{x} = \frac{1}{6} + \frac{1}{10}$

$\frac{1}{x} = \frac{4}{15} \Rightarrow x = 3{,}75$

Clave A

18.



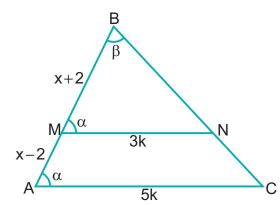
Por propiedad:

$x = \frac{(AB)(DC)}{AB + DC} = \frac{(2)(6)}{2 + 6} = \frac{12}{8}$

$\therefore x = 1{,}5$

Clave E

19.



Por dato: $\overline{MN} \parallel \overline{AC}$

Luego: $\triangle MBN$ a $\triangle ABC$

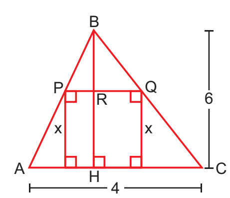
$\Rightarrow \frac{3k}{5k} = \frac{x+2}{2x} \Rightarrow 6x = 5x + 10$

$\therefore x = 10$

Clave D

Resolución de problemas

20. De acuerdo con el enunciado:



$\triangle ABC \sim \triangle PBQ$

$\frac{PQ}{AC} = \frac{BR}{BH}$

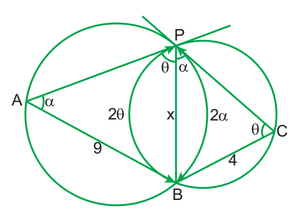
$\frac{x}{4} = \frac{6 - x}{6} \Rightarrow 6x = 24 - 4x$

$10x = 24$

$\therefore x = 2{,}4$

Clave B

21.



Del gráfico: $\triangle APB \sim \triangle PCB$

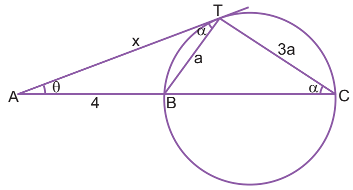
$\frac{PB}{AB} = \frac{BC}{PB} \Rightarrow \frac{x}{9} = \frac{4}{x}$

$x^2 = 36$

$\therefore x = 6$

Clave A

22.



Del gráfico: $m\overset{\frown}{BT} = 2\alpha \Rightarrow m\angle ATB = \alpha$

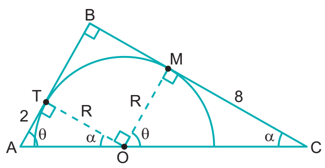
Luego: $\Delta ABT \sim \Delta ATC$

$\dfrac{AB}{TB} = \dfrac{AT}{TC} \Rightarrow \dfrac{4}{a} = \dfrac{x}{3a}$

$\therefore x = 12$

Clave D

23.



Del gráfico: ⊿ATO ~ ⊿OMC

$\Rightarrow \dfrac{R}{2} = \dfrac{8}{R} \Rightarrow R^2 = 16$

$\therefore R = 4$

Clave B

Nivel 3 (página 63) Unidad 3

Comunicación matemática

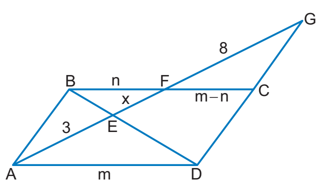
24.

25.

26.

Razonamiento y demostración

27.



$\Delta BEF \sim \Delta AED$

$\dfrac{x}{n} = \dfrac{3}{m} \Rightarrow \dfrac{x}{3} = \dfrac{n}{m}$ ...(1)

$\Delta FGC \sim \Delta AGD$

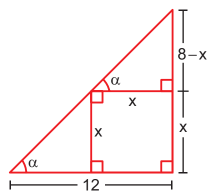
$\dfrac{8}{11+x} = \dfrac{m-n}{m} = 1 - \dfrac{n}{m}$; de (1):

$\dfrac{8}{11+x} = 1 - \dfrac{x}{3} \Rightarrow \dfrac{8}{11+x} = \dfrac{3-x}{3}$

Luego: $x = 1$

Clave D

28.



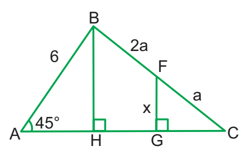
$\dfrac{8-x}{x} = \dfrac{8}{12}$

$24 - 3x = 2x$

$24 = 5x \Rightarrow x = 4,8$

Clave B

29.



$\dfrac{3a}{3\sqrt{2}} = \dfrac{a}{x} \Rightarrow x = \sqrt{2}$

Clave E

30. Se observa que:

$\Delta AQM \sim \Delta NPC$:

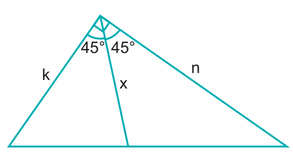
$\dfrac{x}{12} = \dfrac{3}{x} \Rightarrow x^2 = 36$

$\therefore x = 6$

Clave C

Resolución de problemas

31. Según el enunciado:



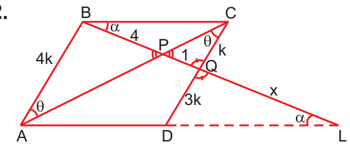
Por propiedad:

$\dfrac{\sqrt{2}}{x} = \dfrac{1}{k} + \dfrac{1}{n} \Rightarrow \dfrac{\sqrt{2}}{x} = \dfrac{n+k}{kn}$

$\dfrac{x}{\sqrt{2}} = \dfrac{kn}{n+k} \quad \therefore x = \dfrac{kn\sqrt{2}}{n+k}$

Clave C

32.



Del gráfico: $\Delta APB \sim \Delta CPQ$

$\Rightarrow \dfrac{AB}{BP} = \dfrac{CQ}{PQ} \Rightarrow \dfrac{AB}{4} = \dfrac{CQ}{1}$

$\therefore AB = 4k \quad \wedge \quad CQ = k$

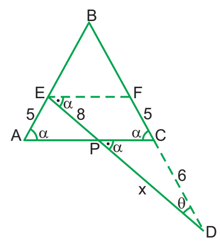
Luego: $\triangle BQC \sim \triangle LQD$

$\Rightarrow \dfrac{QC}{BQ} = \dfrac{DQ}{QL} \Rightarrow \dfrac{k}{5} = \dfrac{3k}{x}$

$\therefore x = 15$

Clave B

33.



Trazamos $\overline{EF}$ // $\overline{AC}$, entonces AEFC resulta un trapecio isósceles.

$\Rightarrow AE = FC = 5$

Luego: $\Delta DPC \sim \Delta DEF$

$\Rightarrow \dfrac{PD}{CD} = \dfrac{ED}{FD} \Rightarrow \dfrac{x}{6} = \dfrac{8+x}{11}$

$11x = 48 + 6x$

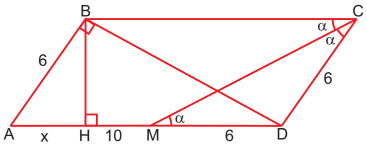
$5x = 48$

$\therefore x = 9,6$

Clave C

APLICAMOS LO APRENDIDO (página 64) Unidad 3

1.



Por propiedad:

$6^2 = x(x + 16)$

$36 = x^2 + 16x$

$0 = x^2 + 16x - 36$

$x \quad + 18$

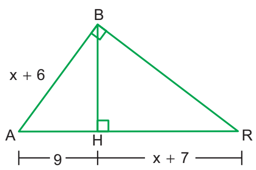
$x \quad - 2$

$\Rightarrow x + 18 = 0 \lor x - 2 = 0$

$x = -18$ (no cumple) $\quad \therefore AH = x = 2$

Clave D

2.



$(x + 6)^2 = 9(x + 16)$

$x^2 + 12x + 36 = 9x + 144$

$x^2 + 3x - 108 = 0$

$x \quad - 9$

$x \quad + 12$

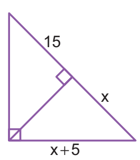
$x - 9 = 0 \lor x + 12 = 0$

$x = 9 \lor \quad x = -12$ (no cumple)

$\Rightarrow x = 9 \quad \therefore HR = 9 + 7 = 16$

Clave B

3.



Por propiedad:

$(x + 5)^2 = x(x + 15)$

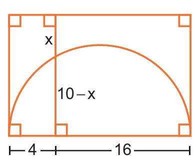
$x^2 + 10x + 25 = x^2 + 15x$

$25 = 5x$

$x = 5$

Clave C

4.



Por propiedad:

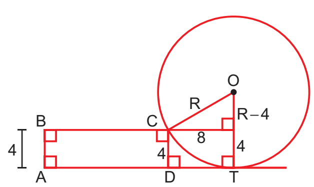
$(10 - x)^2 = 4(16)$

$10 - x = 8$

$x = 2$

Clave B

5.



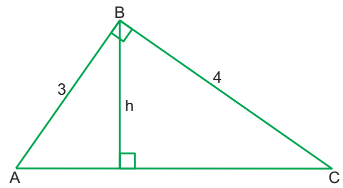
$R^2 = 8^2 + (R - 4)^2$

$R^2 = 64 + R^2 - 8R + 16$

$8R = 80 \Rightarrow R = 10$

Clave A

6.



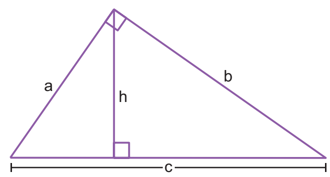
$(AC)^2 = 3^2 + 4^2 = 25 \Rightarrow AC = 5$

$AB \,.\, BC = h \,.\, AC$

$3 \,.\, 4 = h \,.\, 5 \Rightarrow h = \dfrac{12}{5} \quad \therefore \quad h = 2{,}4$

Clave A

7.



Datos: $a + b + c = 10 \quad \ldots (1)$

$ab = 5$

De (1): $(a + b)^2 = (10 - c)^2$

$a^2 + b^2 + 2ab = 100 - 20c + c^2$

$c^2 + 2(5) = 100 - 20c + c^2$

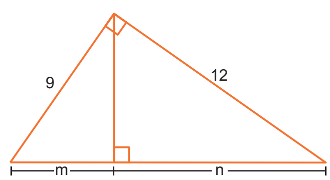
$20c = 90 \Rightarrow c = \dfrac{9}{2}$

Sabemos: $ab = hc$

$5 = h\left(\dfrac{9}{2}\right) \quad \therefore h = \dfrac{10}{9}$

Clave B

8.



$(m + n)^2 = 9^2 + 12^2 = 81 + 144 = 225$

$\Rightarrow m + n = 15 \quad \ldots (1)$

luego: $9^2 = m(m + n)$

$81 = m(15) \Rightarrow m = \dfrac{27}{5} \quad \ldots (2)$

Reemplazando (2) en (1):

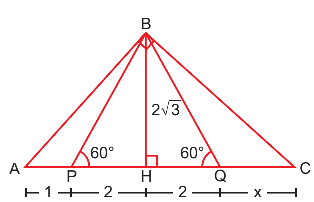
$\dfrac{27}{5} + n = 15 \Rightarrow n = \dfrac{48}{5}$

$\Rightarrow n - m = \dfrac{48}{5} - \dfrac{27}{5} = \dfrac{21}{5}$

$\therefore n - m = 4{,}2$

Clave A

9.

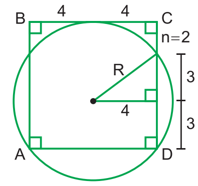


$(2\sqrt{3})^2 = 3(2 + x)$

$4 = 2 + x \Rightarrow x = 2$

Clave A

10.



$4^2 = 8(n)$

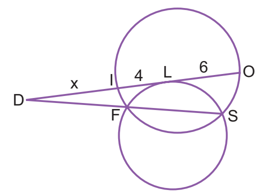
$n = 2$

$R^2 = 4^2 + 3^2$

$R = 5$ m

Clave D

11.



$(x + 4)^2 = (DS)(DF)$

$(x + 10)x = (DS)(DF)$

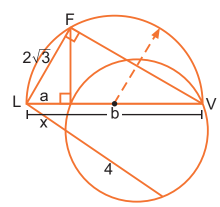
$\Rightarrow (x + 4)^2 = x(10 + x)$

$x^2 + 8x + 16 = 10x + x^2$

$16 = 2x \Rightarrow x = 8$

Clave A

12.



En el $\triangle$LFV:

$(2\sqrt{3})^2 = ba$

$12 = ba$

Teorema de las secantes:

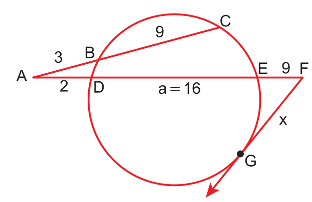
$x(x + 4) = 12$

$\Rightarrow x(x + 4) = 2(2+4)$

$\therefore x = 2$

Clave D

13.



Por el teorema de las secantes:

$AC \,.\, AB = AE \,.\, AD$

$12 \,.\, 3 = 2(2 + a)$

$18 = 2 + a \Rightarrow a = 16$

Por el teorema de la tangente:

$x^2 = 9(9 + 16) = 9 \,.\, 25 = 225$

$\therefore x = 15$

Clave A

14. En la circunferencia de radio R:

$(RG)(GP) = (EG)\,(GH) \quad \ldots (1)$

En la circunferencia de radio r:

$(RG)\,(GP) = (FG)\,(GI) \quad \ldots (2)$

Igualando (1) y (2):

$(EG)\,(GH) = (FG)(GI) \quad \ldots (3)$

Del dato: $EF = 8$; $FG = 2$; $GH = 1$

Reemplazando en (3):

$(8 + 2)\,(1) = (2)\,(1 + x) \Rightarrow x = 4$

Clave E

PRACTIQUEMOS

Nivel 1 (página 66) Unidad 3

Comunicación matemática

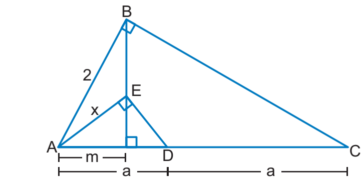
1.

2.

3.

Razonamiento y demostración

4.



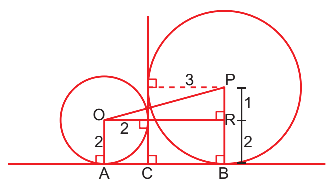
En el ◺ABC: $2^2 = (2a)m$

En el ◺AED: $x^2 = am$

$\Rightarrow 2^2 = 2(x^2)$ $\therefore$ $x = \sqrt{2}$

Clave E

5.



Del gráfico:

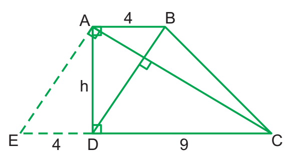
$AC = 2 \wedge CB = 3 \Rightarrow OR = 5$

En el ◺ORP: $(OP)^2 = 5^2 + 1^2$

$\therefore OP = \sqrt{26}$

Clave A

6.



Trazamos $\overline{AE}$ // $\overline{BD}$, entonces se forma el paralelogramo ABDE.

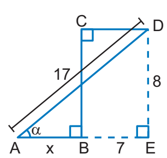
En el ◺ EAC, por relaciones métricas se cumple:

$h^2 = (ED)(DC)$

$h^2 = (4)(9)$ $\therefore$ $h = 6$

Clave B

7.



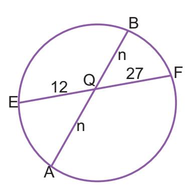
$(x + 7)^2 + 8^2 = 17^2$

$(x + 7)^2 = 15^2$

$\therefore x = 8$

Clave E

8.



Por el teorema de las cuerdas:

$(AQ)(QB) = (EQ)(QF)$

$(n)(n) = (12)(27)$

$n^2 = 324 \Rightarrow n = 18$

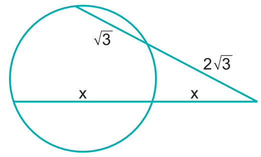
Piden:

$AB = 2n = 2(18)$

$\therefore$ AB = 36

Clave C

9.



Por el teorema de las secantes:

$x(2x) = (2\sqrt{3})(3\sqrt{3})$

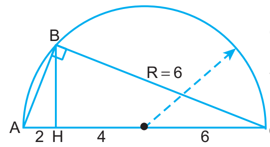
$2x^2 = 18$

$x^2 = 9$ $\therefore$ $x = 3$

Clave D

Resolución de problemas

10.

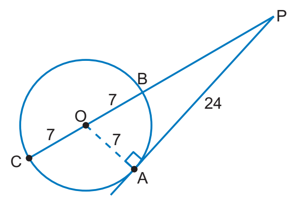


$(AB)^2 = 2(12)$

$\therefore$ $AB = 2\sqrt{6}$ m

Clave C

11.



En el ◺OAP por el teorema de Pitágoras:

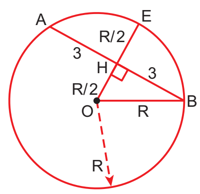
$(OP)^2 = 7^2 + 24^2 = 625 \Rightarrow OP = 25$

Piden: PC = OP + OC = 25 + 7

$\therefore$ PC = 32 m

Clave D

12.



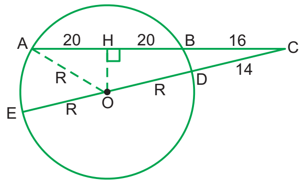
En el ◺OHB por el teorema de Pitágoras:

$R^2 = \left(\frac{R}{2}\right)^2 + 3^2 \Rightarrow R^2 = \frac{R^2}{4} + 9$

$\frac{3R^2}{4} = 9 \Rightarrow R^2 = 12$ $\therefore$ $R = 2\sqrt{3}$

Clave A

13.



En el ◺AHO por el teorema de Pitágoras:

$R^2 = (OH)^2 + 20^2 \Rightarrow (OH)^2 = R^2 - 400$ ...(1)

En el ◺OHC por el teorema de Pitágoras:

$(R + 14)^2 = (OH)^2 + 36^2$

$(OH)^2 = R^2 + 28R - 1100$ ...(2)

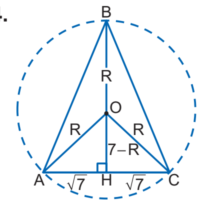
De (1) y (2):

$R^2 - 400 = R^2 + 28R - 1100 \Rightarrow 700 = 28R$

$\therefore R = 25$

Clave B

14.



O es circuncentro:

OA = OB = OC = R

En el ◺AHO:

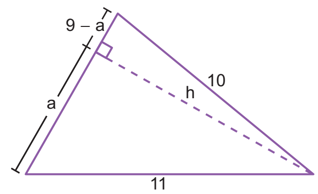
$R^2 = (7 - R)^2 + (\sqrt{7})^2$

$R^2 = 49 - 14R + R^2 + 7$

$14R = 56 \Rightarrow R = 4$

Clave A

15.



La mayor altura en un triángulo es la altura relativa al lado menor.

$11^2 - a^2 = 10^2 - (9 - a)^2$

$21 = a^2 - [81 + a^2 - 18a] \Rightarrow 102 = 18a$

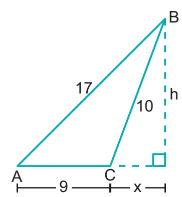
$a = \frac{17}{3}$

Luego: $11^2 = h^2 + a^2 \Rightarrow 121 = h^2 + \frac{289}{9}$

$h^2 = \frac{800}{9}$ $\therefore h = \frac{20\sqrt{2}}{3}$

Clave E

16.



Por naturaleza de un triángulo:

$17^2 > 10^2 + 9^2 \Rightarrow \angle C$ es obtuso

$h^2 = 10^2 - x^2 = 17^2 - (x + 9)^2$

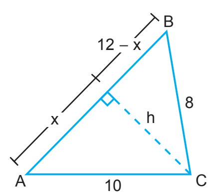
$17^2 - 10^2 = (x + 9)^2 - x^2$

$(27)(7) = (2x + 9)(9)$

$21 = 2x + 9$ $\therefore x = 6$

Clave E

17.



Por naturaleza de un triángulo.

$12^2 < 8^2 + 10^2 \Rightarrow \angle C$ es agudo

$h^2 = 8^2 - (12 - x)^2 = 10^2 - x^2$

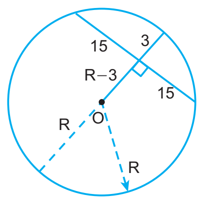
$x^2 - (12 - x)^2 = 10^2 - 8^2$

$(12)(2x - 12) = (2)(18)$

$2x - 12 = 3$ $\therefore x = 15/2$

Clave E

18.

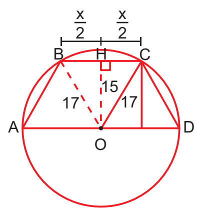


Por el teorema de cuerdas:

$15(15) = (2R - 3)3 \qquad \therefore\ R = 39$ cm

Clave B

19.



En el $\triangle$BOC isósceles: $BH = HC = \dfrac{x}{2}$

En el ⊿OHC: $\left(\dfrac{x}{2}\right)^2 = 17^2 - 15^2$

$\left(\dfrac{x}{2}\right)^2 = 64 \Rightarrow \dfrac{x}{2} = 8$

$\therefore x = 16$ cm

Clave B

Nivel 2 (página 67) Unidad 3

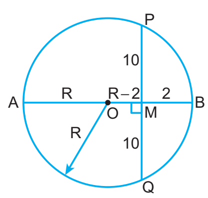
Comunicación matemática

20.

21.

Razonamiento y demostración

22.



Por el teorema de las cuerdas:

$(AM)(MB) = (PM)(MQ)$

$(2R - 2)(2) = (10)(10)$

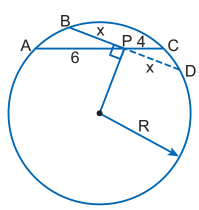
$2R - 2 = 50$

$2R = 52$

$\therefore R = 26$

Clave E

23.



Por el teorema de las cuerdas:

$(AP)(PC) = (BP)(PD)$

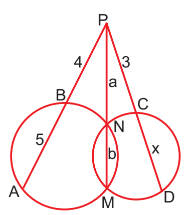
$(6)(4) = (x)(x)$

$24 = x^2$

$\therefore x = 2\sqrt{6}$

Clave D

24.



Por el teorema de las secantes:

Para $\overline{PA}$ y $\overline{PM}$: $\quad (9)(4) = (a + b)(a)$ ...(1)

Para $\overline{PM}$ y $\overline{PD}$: $(x + 3)(3) = (a + b)(a)$ ...(2)

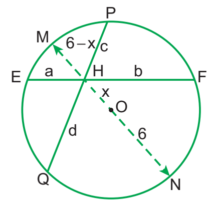
De (1) y (2): $(x + 3)(3) = (9)(4)$

$x + 3 = 12$

$\therefore x = 9$

Clave C

25.



Por dato: $abcd = 625$

Por el teorema de las cuerdas:

$(6 - x)(6 + x) = ab$ ...(1)

$(6 - x)(6 + x) = cd$ ...(2)

Multiplicando (1) y (2):

$(6^2 - x^2)^2 = abcd$

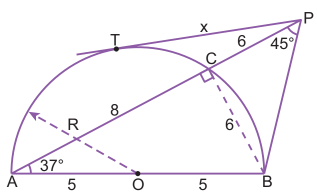
$(36 - x^2)^2 = 625$

$36 - x^2 = 25$

$\Rightarrow x^2 = 11 \qquad \therefore\ x = \sqrt{11}$

Clave E

26.



Por dato: $R = 5 \Rightarrow AB = 10$

El ⊿ACB es notable de 37° y 53°:

$\Rightarrow CB = 6 \wedge AC = 8$

El ⊿PCB es notable de 45°.

$\Rightarrow CB = CP = 6$

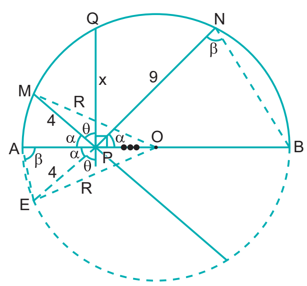
Por el teorema de la tangente:

$x^2 = (8 + 6)(6)$

$x^2 = (14)(6) \qquad \therefore x = 2\sqrt{21}$

Clave D

27.



El $\triangle OMP \cong \triangle OEP$ (caso LLL) $\Rightarrow PM = PE = 4$

Luego: $\triangle APE \sim \triangle NPB$

$\dfrac{PE}{AP} = \dfrac{PB}{NP} \Rightarrow \dfrac{4}{AP} = \dfrac{PB}{9}$

Entonces: $AP \cdot PB = 36$

En la semicircunferencia AQB por uno de los teoremas adicionales:

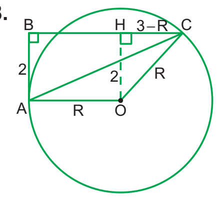
$x^2 = AP \cdot PB \Rightarrow x^2 = 36$

$\therefore x = 6$

Clave A

Resolución de problemas

28.



En el ⊿OHC:

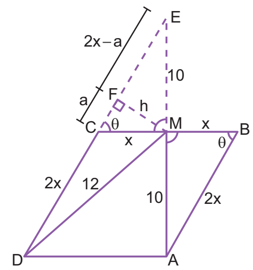
$2^2 + (3 - R)^2 = R^2$

$4 + 9 - 6R + R^2 = R^2$

$\therefore R = \dfrac{13}{6}$

Clave C

29.



Prolongamos $\overline{AM}$ y $\overline{DC}$ intersecándose en el punto E.

$\triangle BMA \cong \triangle CME$ (caso ALA)

$\Rightarrow EM = 10 \wedge CE = 2x$

En el ⊿CFM: $a^2 + h^2 = x^2$

En el ⊿EFM: $10^2 = (2x - a)^2 + h^2$ ... (1)

En el ⊿DFM: $12^2 = (2x + a)^2 + h^2$ ... (2)

Sumando (1) y (2):

$244 = 2h^2 + 2(4x^2 + a^2)$

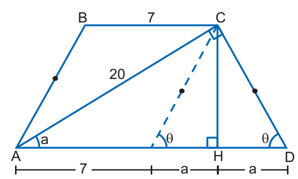
$244 = 2(h^2 + a^2) + 8x^2$

$244 = 2x^2 + 8x^2$

$\Rightarrow x = \sqrt{\dfrac{122}{5}} \qquad \therefore AB = 2x = 2\sqrt{\dfrac{122}{5}}$

Clave C

30.



En el ⊿ACD por relaciones métricas:

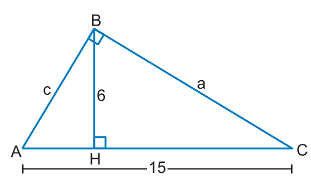
$(AC)^2 = AH \cdot AD$

$20^2 = (7 + 2a)(7 + a) \Rightarrow a = 9$

$\therefore AD = 7 + 2a = 7 + 2(9) = 25$

Clave A

31.



Piden: $a - c$

Por el teorema de Pitágoras:

$a^2 + c^2 = 15^2 \Rightarrow a^2 + c^2 = 225$ ...(1)

En el ⊿ABC, por el producto de catetos:

$(a)(c) = (6)(15)$

$ac = 90 \Rightarrow 2ac = 180$ ...(2)

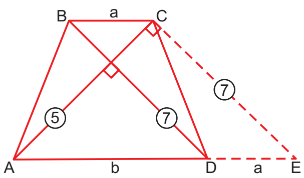
Restando (1) y (2):

$a^2 + c^2 - 2ac = 225 - 180$

$(a - c)^2 = 45 \qquad \therefore a - c = 3\sqrt{5}$ m

Clave D

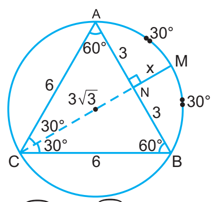
32.



Trazamos $\overline{CE}$ // $\overline{BD}$, entonces BDEC es un paralelogramo.
En el ⊿ACE por el teorema de Pitágoras:
$(a+b)^2 = 5^2 + 7^2 = 74 \Rightarrow a + b = \sqrt{74}$
Piden: la mediana del trapecio ABCD (m)
$\Rightarrow m = \dfrac{a+b}{2} = \dfrac{\sqrt{74}}{2}$ $\qquad \therefore m = \dfrac{\sqrt{74}}{2}$

Clave B

33.



Por dato: $m\overset{\frown}{AM} = m\overset{\frown}{MB}$
En el ⊿ANC notable de 30° y 60°:
$AN = 3 \wedge CN = 3\sqrt{3}$
Por el teorema de las cuerdas:
$(CN)(NM) = (AN)(NB)$
$(3\sqrt{3})(x) = (3)(3)$ $\qquad \therefore x = \sqrt{3}$

Clave C

Nivel 3 (página 68) Unidad 3

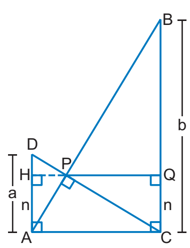
Comunicación matemática

34.

35.

Razonamiento y demostración

36.

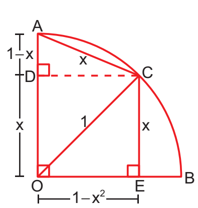


En el ⊿CPB, por relaciones métricas se cumple:
$(CP)^2 = (n)(b)$ ...(1)
En el ⊿APD, por relaciones métricas se cumple:
$(AP)^2 = (n)(a)$ ...(2)

Por dato: $(AD + BC)(QC) = 36$
$\Rightarrow (a + b)(n) = 36$
Sumando (1) y (2):
$(CP)^2 + (AP)^2 = n(b) + n(a) = n(a + b)$
$\Rightarrow (CP)^2 + (AP)^2 = 36$
Por el teorema de Pitágoras en el ⊿APC:
$(AC)^2 = (AP)^2 + (CP)^2$
$(AC)^2 = 36$ $\qquad \therefore AC = 6$

Clave C

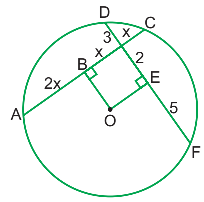
37.



En el ⊿OEC: $OE = \sqrt{1-x^2}$
Del gráfico: $OE = DC$
En el ⊿ADC: $(1 - x)^2 + (\sqrt{1-x^2})^2 = x^2$
$2 - 2x = x^2$
$\therefore x = \sqrt{3} - 1$

Clave E

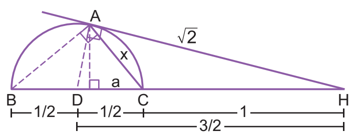
38.



$\overline{OB} \perp \overline{BC} \Rightarrow AB = BC = 2x$
$\overline{OE} \perp \overline{DE} \Rightarrow DE = EF = 5$
Por el teorema de cuerdas:
$3(7) = x(3x) \Rightarrow x^2 = 7 \Rightarrow x = \sqrt{7}$

Clave D

39.



Teorema de la tangente:
$(AH)^2 = 2(1) \Rightarrow AH = \sqrt{2}$
En el ⊿DAH: $(\sqrt{2})^2 = \dfrac{3}{2}(a+1)$ ... (1)
En el ⊿BAC: $x^2 = 1(a)$ ... (2)
De (1) y (2): $2 = \dfrac{3}{2}(x^2 + 1)$
$\dfrac{1}{3} = x^2$ $\qquad \therefore x = \dfrac{\sqrt{3}}{3}$

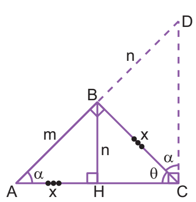
Clave C

40. Vemos que $\Delta BPA \sim \Delta CQA \Rightarrow \dfrac{AB}{AC} = \dfrac{BP}{QC}$
$\therefore AB.QC = BP.AC \Rightarrow AB.QC = BP.\dfrac{AP}{2}$
$\Rightarrow BP.AP = 22$ ...(1)
del dato: $(BP-AP)^2 = 100$; desarrollando el binomio
$BP^2 + AP^2 - 2(BP)(AP) = 100$;
pero $BP^2 + AP^2 = AB^2$, reemplazando:
$\therefore AB^2 - 2(22) = 100 \Rightarrow AB^2 = 144 \Rightarrow AB = 12$

Clave E

Resolución de problemas

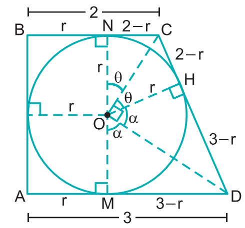
41.



Por dato: $mn = 12$
Prolongamos $\overline{AB}$ tal que: $BH = BD$
Entonces: ⊿AHB ≅ ⊿CBD (caso LLL)
$\Rightarrow m\angle BAH = m\angle BCD = \alpha$
En el ⊿ABC: $\alpha + \theta = 90° \Rightarrow m\angle ACD = 90°$
En el ⊿ACD, se cumple:
$x^2 = m\,.\,n \Rightarrow x^2 = 12$ $\qquad \therefore x = 2\sqrt{3}$

Clave B

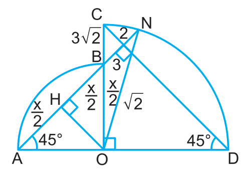
42.



Del gráfico:
$2\theta + 2\alpha = 180° \Rightarrow \theta + \alpha = 90°$
En el ⊿COD, se cumple:
$r^2 = (2 - r)(3 - r) \Rightarrow r^2 = 6 - 5r + r^2$
$5r = 6$ $\qquad \therefore r = \dfrac{6}{5}$

Clave B

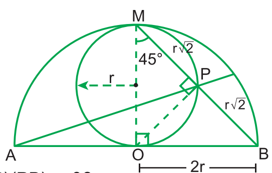
43.



En el ⊿NHO:
$\left(\dfrac{x}{2}\sqrt{2} + 3\sqrt{2}\right)^2 = \left(\dfrac{x}{2}\right)^2 + \left(\dfrac{x}{2} + 5\right)^2$
$\dfrac{x^2 + 36 + 12x}{2} = \dfrac{x^2}{4} + \dfrac{x^2}{4} + 25 + 5x$
$\dfrac{x^2 + 36 + 12x}{2} = \dfrac{x^2}{2} + 5x + 25$
$36 + 12x = 10x + 50$
$2x = 14$
$x = 7$

Clave A

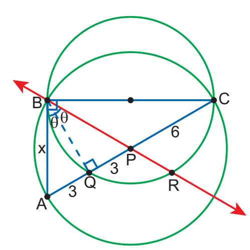
44.



$(MP)(PB) = 32$
$(r\sqrt{2})^2 = (MP)(PB)$
$\Rightarrow 2r^2 = 32 \Rightarrow r = 16 \Rightarrow r = 4$ $\qquad \therefore 2r = 8$

Clave E

45.



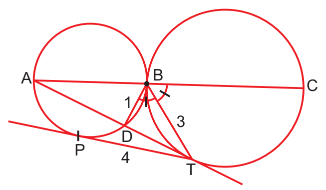
Del gráfico:
$m\angle ABC = m\angle BQC = 90°$
Luego: $m\angle ABQ = m\angle QBP = \theta$
$\therefore \Delta ABP$ es isósceles $\Rightarrow AQ = QP = 3$
$\therefore AB^2 = (AQ)(AC)$
$x^2 = (3)(12) \Rightarrow x = 6$

Clave A

RELACIONES MÉTRICAS EN TRIÁNGULOS OBLICUÁNGULOS

APLICAMOS LO APRENDIDO (página 70) Unidad 3

1.



Del gráfico, $\overline{BT}$ es bisectriz exterior.

En el ΔABT (teorema de la bisectriz exterior).

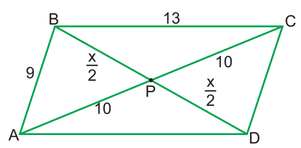
$(BT)^2 = (DT)(AT) - (AB)(BD)$

$3^2 = PT^2 - (AB)(1)$

$9 = 4^2 - (AB)$ $\qquad \therefore$ AB = 7

Clave A

2.



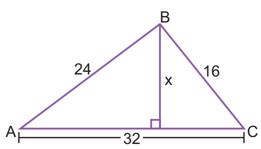
En el ΔABC (T. de la mediana):

$$AB^2 + BC^2 = 2BP^2 + \frac{AC^2}{2}$$

$$9^2 + 13^2 = 2\left(\frac{x}{2}\right)^2 + \frac{20^2}{2} \qquad \therefore\ x = 10$$

Clave E

3.



En el ΔABC (T. de Herón):

$$x = \frac{2}{32}\sqrt{p(p-24)(p-16)(p-32)}$$

$$p = \frac{24+16+32}{2}$$

p = 36

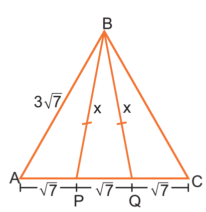
Luego :

$$x = \frac{2}{32}\sqrt{36(36-24)(36-16)(36-32)}$$

$\therefore\ x = 3\sqrt{15}$

Clave B

4.



Del gráfico; ΔPBQ isósceles

En el ΔABQ (T. de la mediana):

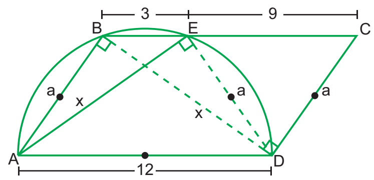
$$AB^2 + BQ^2 = 2BP^2 + \frac{AQ^2}{2}$$

$$(3\sqrt{7})^2 + x^2 = 2x^2 + \frac{(2\sqrt{7})^2}{2}$$

$x^2 = 49$ $\qquad \therefore x = 7$

Clave A

5.



ABED trapecio isósceles:

$\Rightarrow$ AE = BD = x $\wedge$ AB = ED = a

En el triángulo BDC (T. de Stewart):

$(BD)^2(EC) + (DC)^2(BE) = (ED)^2(BC) + (BE)(EC)(BC)$

$x^2(9) + a^2(3) = a^2(12) + (3)(9)(12)$

$9x^2 = 9a^2 + (3)(9)(12)$

$x^2 - a^2 = 36 \qquad ...\ (1)$

Además:

$AB^2 + BD^2 = AD^2$

$a^2 + x^2 = 12^2 \qquad ...\ (2)$

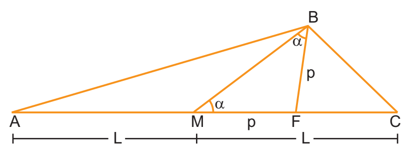
De (1) y (2):

$2x^2 = 36 + 144$

$x^2 = 90$ $\qquad \therefore\ x = 3\sqrt{10}$

Clave B

6.



En el ABC (T. de la bisectriz):

$(BF)^2 = (AB)(BC) - (AF)(FC)$

$p^2 = 25 - (L + p)(L - p)$

$p^2 = 25 - L^2 + p^2$

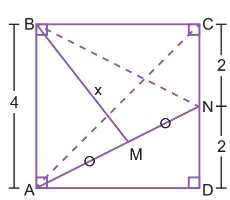
$L^2 = 25$

L = 5

$\therefore$ AC = 2L = 10 m

Clave B

7.



$AC = 4\sqrt{2} \Rightarrow AB = BC = CD = AD = 4$

Del gráfico ΔAND y ΔBCN notables de

$\frac{53^\circ}{2}$ y $\frac{127^\circ}{2}$

$\Rightarrow AN = BN = 2\sqrt{5}$

En el ΔABN (T. de la mediana):

$$AB^2 + BN^2 = 2BM^2 + \frac{AN^2}{2}$$

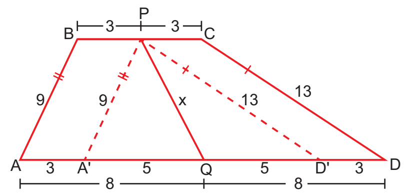
$$4^2 + (2\sqrt{5})^2 = 2x^2 + \frac{(2\sqrt{5})^2}{2}$$

$16 + 20 = 2x^2 + 10$

$\therefore\ x = \sqrt{13}$ u

Clave E

8.



Trazamos $\overline{PA'} \parallel \overline{AB}$ y $\overline{PD'} \parallel \overline{CD}$

En el ΔPA'D' (T. de la mediana):

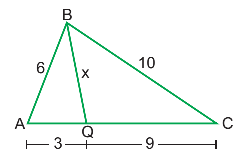
$$A'P^2 + D'P^2 = 2PQ^2 + \frac{(A'D')^2}{2}$$

$$9^2 + 13^2 = 2x^2 + \frac{(10)^2}{2}$$

$\therefore$ x = 10

Clave C

9.



Teorema de Stewart:

$12(x^2) = (10^2)3 + (6^2)9 - 3(9)(12)$

$12x^2 = 300 + 324 - 324$

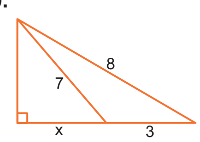
$12x^2 = 300$

$x^2 = 25$

x = 5

Clave C

10.



2.° teorema de Euclides:

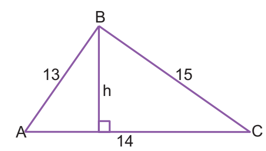
$8^2 = 7^2 + 3^2 + 2(3x)$

64 = 58 + 6x

$\therefore$ x = 1

Clave A

11.



Tenemos: $p = \dfrac{13+15+14}{2} = 21$

Por el teorema de Herón:

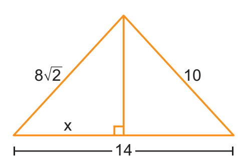
$$h = \frac{2}{14}\sqrt{21(21-13)(21-14)(21-15)}$$

$$h = \frac{1}{7}\sqrt{21\cdot 8\cdot 7\cdot 6}$$

$$h = \frac{1}{7}(84) \qquad \therefore\ h = 12$$

Clave D

12.



1.^{er} teorema de Euclides:

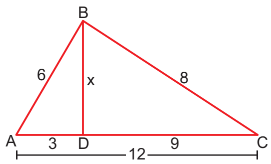
$10^2 = (8\sqrt{2})^2 + (14)^2 - 2(14)x$

$100 = 128 + 196 - 28x$

$28x = 224 \qquad \therefore x = 8$

Clave B

13.



Por el teorema de Stewart:

$6^2(9) + 8^2(3) = x^2(12) + 3(9)(12)$

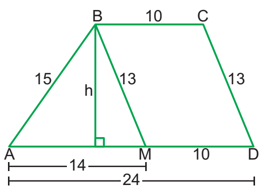
$516 = 12x^2 + 324$

$192 = 12x^2$

$\therefore x = 4$

Clave A

14. Piden: h



Se traza $\overline{BM}$ // $\overline{CD}$; de donde MBCD: es un paralelogramo.

$\Rightarrow BM = 13$ y $AM = 14$

En el $\triangle ABM$ por el teorema del cálculo de la altura, tenemos:

$h = \dfrac{2}{14}\sqrt{p(p-13)(p-14)(p-15)}$

Donde: $p = \dfrac{13+14+15}{2} \Rightarrow p = 21$

Reemplazando:

$h = \dfrac{2}{14}\sqrt{21(21-13)(21-14)(21-15)}$

$h = \dfrac{2}{14}\sqrt{21(8)(7)(6)}$

$\therefore h = 12$

Clave C

PRACTIQUEMOS
Nivel 1 (página 72) Unidad 3

Comunicación matemática

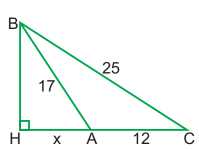
1.

2.

3.

Razonamiento y demostración

4. De la figura:



Por el teorema de Euclides:

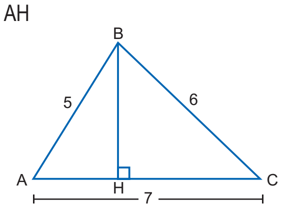
$25^2 = 17^2 + 12^2 + 2 \cdot 12 \cdot x$

$625 = 289 + 144 + 24x$

$192 = 24x \Rightarrow x = 8$

Clave C

5. Piden: AH



$BH = \dfrac{2}{7}\sqrt{9(9-5)(9-6)(9-7)}$

$BH = \dfrac{2}{7}\sqrt{9(4)(3)(2)} = \dfrac{2 \times 3 \times 2}{7}\sqrt{6}$

$BH = \dfrac{12}{7}\sqrt{6} \Rightarrow (BH)^2 = \dfrac{144 \times 6}{49}$

$(AH)^2 + (BH)^2 = 25$

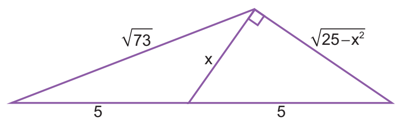
$(AH)^2 + \dfrac{144 \times 6}{49} = 25$

$(AH)^2 = 25 - \dfrac{144 \times 6}{49} = \dfrac{25 \times 49 - 144 \times 6}{49}$

$(AH)^2 = \dfrac{361}{49} \Rightarrow AH = \dfrac{19}{7}$

Clave C

6.



$(\sqrt{73})^2 + (\sqrt{25-x^2})^2 = 2x^2 + 2(5)^2$

$73 + 25 - x^2 = 2x^2 + 50$

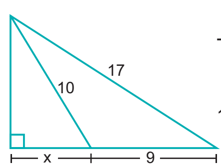
$48 = 3x^2$

$16 = x^2$

$x = 4$

Clave D

7.



Teorema de Euclides:

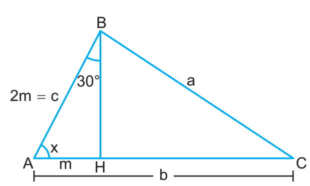
$17^2 = 10^2 + 9^2 + 2(9)x$

$108 = 18x \Rightarrow 6 = x$

Clave B

Resolución de problemas

8.



Por dato: $a^2 = b^2 + c^2 - bc \quad ...(1)$

Por el primer teorema de Euclides:

$a^2 = b^2 + c^2 - 2bm \quad ...(2)$

De (1) y (2):

$b^2 + c^2 - bc = b^2 + c^2 - 2bm$

$bc = 2bm$

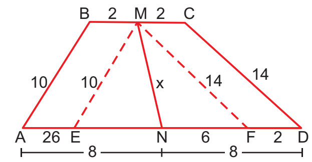
$\Rightarrow c = 2m$

Entonces, el ⊿BHA resulta notable de 30° y 60°.

$\therefore x = 60°$

Clave B

9.



Trazamos $\overline{ME}$ // $\overline{BA}$ y $\overline{MF}$ // $\overline{CD}$, entonces se forman los paralelogramos ABME y MCDF.

En el $\triangle EMF$ por el teorema de la mediana:

$10^2 + 14^2 = 2(x)^2 + \dfrac{(12)^2}{2}$

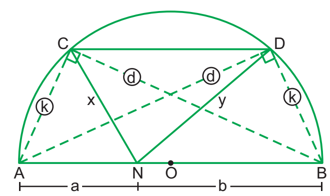
$296 = 2x^2 + 72$

$x^2 = 112$

$\therefore x = 4\sqrt{7}$

Clave C

10.



Por dato: $\overline{CD}$ // $\overline{AB}$, entonces ABCD es un trapecio isósceles.

Además: $a^2 + b^2 = 100$

En el ⊿ACB por el teorema de Stewart:

$k^2b + d^2a = x^2(a+b) + ab(a+b) \quad ...(1)$

En el ⊿ADB por el teorema de Stewart:

$k^2a + d^2b = y^2(a+b) + ba(a+b) \quad ...(2)$

Sumando y agrupando (1) y (2):

$(a+b)(k^2+d^2) = (a+b)(x^2+y^2) + 2ab(a+b)$

$\Rightarrow k^2 + d^2 = x^2 + y^2 + 2ab \quad ...(3)$

En el ⊿ACB por el teorema de Pitágoras:

$k^2 + d^2 = (a+b)^2 \quad ...(4)$

Reemplazando (4) en (3):

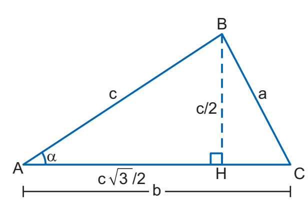
$a^2 + 2ab + b^2 = x^2 + y^2 + 2ab$

$\Rightarrow x^2 + y^2 = a^2 + b^2$

$\therefore x^2 + y^2 = 100$

Clave D

11.



Por dato:

$a^2 = b^2 + c^2 - bc\sqrt{3} \quad ...(1)$

En el $\triangle ABC$, por el primer teorema de Euclides:

$a^2 = b^2 + c^2 - 2b(AH)$ ...(2)

De (1) y (2):

$b^2 + c^2 - bc\sqrt{3} = b^2 + c^2 - 2b(AH)$

$bc\sqrt{3} = 2b(AH)$

$\Rightarrow AH = \dfrac{c\sqrt{3}}{2}$

En el ⊿AHB, por el teorema de Pitágoras:

$BH = \dfrac{c}{2}$

Entonces el ⊿BHA resulta ser notable de 30° y 60°.

$\therefore\ \alpha = 30°$

Clave B

Nivel 2 (página 73) Unidad 3

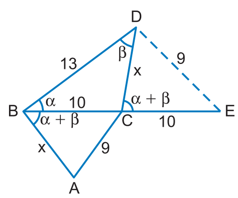
Comunicación matemática

12.

13.

Razonamiento y demostración

14.



$\triangle ABC \cong \triangle DCE$ (caso LAL)

Teorema de la mediana:

$13^2 + 9^2 = 2x^2 + 2(10)^2$

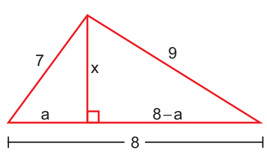
$250 = 2x^2 + 200$

$50 = 2x^2$

$25 = x^2 \qquad \therefore\ x = 5$

Clave C

15.



$7^2 - a^2 = 9^2 - (8 - a)^2$

$49 - a^2 = 81 - 64 - a^2 + 16a$

$32 = 16a$

$2 = a$

$x^2 = 7^2 - a^2$

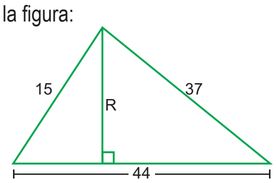
$x^2 = 49 - 2^2$

$x^2 = 45$

$x = 3\sqrt{5}$

Clave C

16. De la figura:



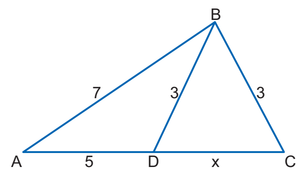
$R = \dfrac{2}{44}\sqrt{48(48-15)(48-37)(48-44)}$

$R = \dfrac{2}{44}\sqrt{48(33)(11)(4)}$

$\therefore\ R = \dfrac{2}{44}\sqrt{69\ 696} = 12$

Clave C

17.



En el $\triangle ABC$ por el teorema de Stewart:

$7^2(x) + 3^2(5) = 3^2(5 + x) + 5(x)(5 + x)$

$49x + 45 = 45 + 9x + 25x + 5x^2$

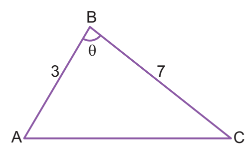
$15x = 5x^2$

$15 = 5x \qquad \therefore\ x = 3$

Clave C

Resolución de problemas

18.



Por dato: $\theta < 90°$

Por desigualdad triangular:

$7 - 3 < AC < 7 + 3 \Rightarrow 4 < AC < 10$ ...(1)

Pero también debe verificar:

$(AC)^2 < 3^2 + 7^2$

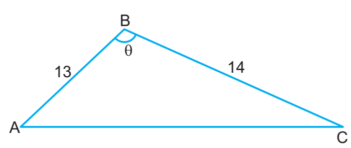
$(AC)^2 < 58 \Rightarrow AC < 7{,}6$ ...(2)

De (1) y (2): $4 < AC < 7{,}6$

Por lo tanto, el máximo valor entero de AC es 7.

Clave C

19.



Por dato: $\theta > 90°$, entonces el $\triangle ABC$ es obtusángulo.

Luego por la naturaleza de un triángulo:

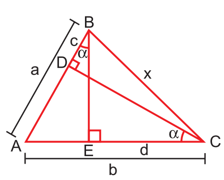
$(AC)^2 > 13^2 + 14^2$

$(AC)^2 > 365 \Rightarrow AC > 19{,}1$

Por lo tanto el mínimo valor entero de AC es 20.

Clave B

20.



$bd = 88$

$ac = 108$

Por propiedad:

$x^2 = ac + bd$

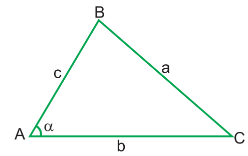
$x^2 = 108 + 88$

$x^2 = 196$

$\therefore x = 14$

Clave D

21.



$a^2 = b^2 + c^2 + bc\sqrt{2}$

$a^2 = b^2 + c^2 - 2bc(\cos\alpha)$ (–)

$0 = bc\sqrt{2} + 2bc(\cos\alpha)$

$0 = 2\cos\alpha + \sqrt{2}$

$-\dfrac{\sqrt{2}}{2} = \cos\alpha$

$\therefore \alpha = 135°$

Clave E

Nivel 3 (página 73) Unidad 3

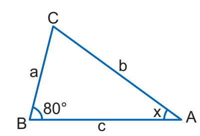
Comunicación matemática

22.

23.

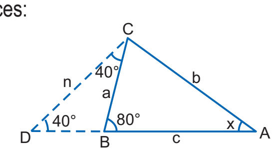
Razonamiento y demostración

24. Del gráfico:



Por dato: $b^2 = a^2 + ac$

Entonces:



Prolongamos $\overline{AB}$ tal que: $CB = BD = a$

En el $\triangle DCA$ por el teorema de Stewart:

$n^2c + b^2a = a^2(a + c) + (a + c)ac$

$n^2c + b^2a = (a + c)\underbrace{(a^2 + ac)}_{b^2}$

$n^2c + b^2a = b^2a + cb^2$

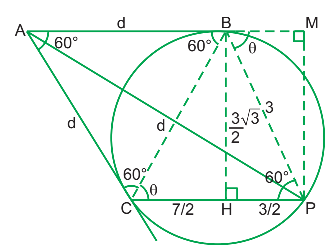
$n^2c = cb^2 \Rightarrow n = b$

Luego el $\triangle DCA$ resulta isósceles:

$\therefore x = 40°$

Clave B

25.



Del gráfico: $m\overset{\frown}{BC} = 120° \Rightarrow m\angle BPC = 60°$

En el ⊿BHC por ley de cosenos: $d = \sqrt{19}$

⊿CHB ~ ⊿BMP

$\dfrac{BM}{3} = \dfrac{7/2}{d} \Rightarrow BM = \dfrac{21}{2d} = \dfrac{21}{2\sqrt{19}}$

En el $\triangle ABP$ por el segundo teorema de Euclides:

$(AP)^2 = d^2 + 3^2 + 2d(BM)$

$(AP)^2 = \sqrt{19}^{\,2} + 9 + 2(\sqrt{19})\left(\dfrac{21}{2\sqrt{19}}\right)$

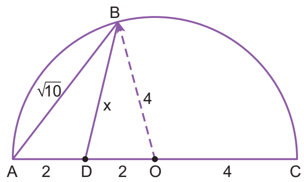
$(AP)^2 = 19 + 9 + 21$

$\Rightarrow (AP)^2 = 49$

$\therefore AP = 7$

Clave B

26.



Por dato: $\overline{AC}$ es diámetro

Entonces: $AO = OC = OB = 4$

En el $\triangle ABO$ por el teorema de la mediana:

$(\sqrt{10})^2 + (4)^2 = 2(x)^2 + \dfrac{(4)^2}{2}$

$10 + 16 = 2x^2 + 8$

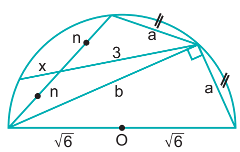
$x^2 = 9$

$\therefore x = 3$

Clave C

Resolución de problemas

27.



$a^2 + b^2 = (2\sqrt{6})^2 = 24$

Teorema de la mediana:

$a^2 + b^2 = 2(3^2) + 2n^2$

$24 = 18 + 2n^2$

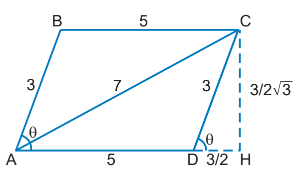
$6 = 2n^2 \Rightarrow 3 = n^2$

Teorema de las cuerdas:

$3x = n^2 \Rightarrow 3x = 3 \Rightarrow x = 1$

Clave C

28.



En el $\triangle ADC$ por el segundo teorema de Euclides:

$7^2 = 5^2 + 3^2 + 2(5)(DH)$

$10(DH) = 15 \Rightarrow DH = \dfrac{3}{2}$

En el ⊿DHC por el teorema de Pitágoras:

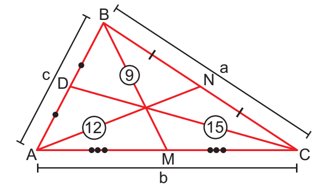
$CH = \dfrac{3\sqrt{3}}{2}$

Entonces el ⊿CHD resulta notable de 30° y 60°.

$\therefore \theta = 60°$

Clave D

29.



En el $\triangle ABC$ por el teorema de la mediana:

$a^2 + c^2 = 2(9)^2 + \dfrac{b^2}{2}$ ...(1)

$a^2 + b^2 = 2(15)^2 + \dfrac{c^2}{2}$ ...(2)

$b^2 + c^2 = 2(12)^2 + \dfrac{a^2}{2}$ ...(3)

Sumando las expresiones (1), (2) y (3):

$2(a^2 + b^2 + c^2) = 900 + \dfrac{a^2 + b^2 + c^2}{2}$

$\Rightarrow a^2 + b^2 + c^2 = 600$ ...(4)

El menor lado es aquel hacia el cual está dirigida la mayor mediana, o sea $\overline{AB}$

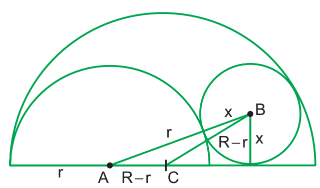
Reemplazando (2) en (4):

$2(15)^2 + \dfrac{c^2}{2} + c^2 = 600$

Resolviendo: $c = 10$

Clave C

30.



En el $\triangle ABC$, por el teorema de Herón:

$p = \dfrac{r + x + R - x + R - r}{2} \Rightarrow p = R$

$x = \dfrac{2}{R - r}\sqrt{R(R - (r + x))(R - (R - r))(R - (R - x))}$

$(R - r)^2x^2 = 4R(R - r - x)(r)x$

$(R - r)^2x = 4Rr(R - r - x)$

$(R - r)^2x + 4Rrx = 4Rr(R - r)$

$x(R^2 - 2Rr + r^2 + 4Rr) = 4Rr(R - r)$

$x(R + r)^2 = 4Rr(R - r)$

$\therefore x = \dfrac{4Rr(R - r)}{(R + r)^2}$

Clave A

31. Se debe cumplir:

$4 - 3 < x < 3 + 4$

$1 < x < 7$

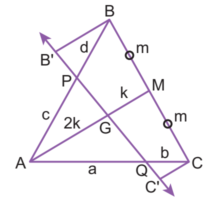
$x \in \{2; 3; 4; 5; 6\}$

$\downarrow$ Rectángulo; $\rightarrow$ Obtusángulo

Clave E

MARATÓN MATEMÁTICA (página 75)

1.



G: baricentro.

$\Rightarrow AG = 2(GM) = 2k$

Trazamos $\overline{CC'} \,//\, \overline{AM} \,//\, \overline{BB'}$

$\triangle CQC' \sim \triangle AGQ$

$\Rightarrow \dfrac{CC'}{b} = \dfrac{2k}{c} \Rightarrow CC' = \dfrac{2kb}{c}$ ... (α)

$\triangle BPB' \sim \triangle APG$

$\Rightarrow \dfrac{BB'}{d} = \dfrac{2k}{c} \Rightarrow BB' = \dfrac{2kd}{c}$ ... (β)

En el trapecio BB'C'C, se cumple:

$k = \dfrac{BB' + CC'}{2}$

Reemplazando (α) y (β) en la relación:

$\Rightarrow ca = da + bc$ ... (γ)

Por dato sabemos:

$AP(QC) + PB(AQ) = 20 \Rightarrow c(b) + d(a) = 20$

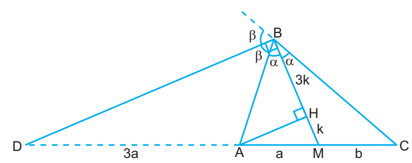
Reemplazando (γ) en la relación

$ca = da + cb = 20$

Piden: $(AP)(AQ) = ca = 20$

Clave D

2.



Trazamos la bisectriz exterior $\overline{BD}$:

$\Rightarrow m\angle DBM = 90° \Rightarrow \overline{DB} \,//\, \overline{AH}$

Por el teorema de Thales en el $\triangle BDM$:

$\dfrac{DA}{AM} = \dfrac{BH}{HM} \Rightarrow DA = 3a$

El $\triangle ABC$ con la bisectriz exterior $\overline{BD}$ forman una Cuaterna armónica:

$\Rightarrow (DA)(MC) = (AM)(CD)$

$3ab = a(4a + b) \Rightarrow b = 2a$ ... (α)

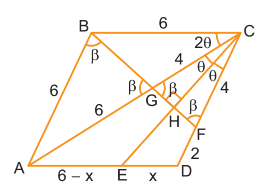
Nos piden $\dfrac{AM}{MC}$:

$\dfrac{AM}{MC} = \dfrac{a}{b}$, reemplazando (α).

$\dfrac{AM}{MC} = \dfrac{a}{b} = \dfrac{a}{2a} = 0{,}5$

Clave E

3.



ABCD es un rombo:

$\Rightarrow AB = BC = CD = AD = 6$

$\Rightarrow m\angle BCA = m\angle ACD = 2\theta \Rightarrow m\angle ACE = \theta$

Dentro del $\triangle GCF$:

$\triangle CHG \cong \triangle CHF$ (caso ALA)

$\Rightarrow GC = FC = 4$

$\Rightarrow m\angle CGF = m\angle GFC = \beta$

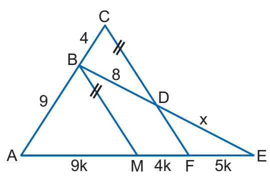
En el $\triangle BAG$: $AB = AG = 6$

En el $\triangle ACD$, por el teorema de la bisectriz interior.

$$\frac{AC}{AE} = \frac{CD}{BD} \Rightarrow \frac{10}{6-x} = \frac{6}{x} \Rightarrow x = \frac{9}{4}$$

Clave D

4.



En el $\triangle ACF$:

Por el Teorema de Thales:

$$\Rightarrow \frac{AB}{BC} = \frac{AM}{MF} = \frac{9}{4} \Rightarrow AM = 9k \text{ y } MF = 4k$$

Por dato del problema:

$AM = ME \Rightarrow AM = MF + FE$

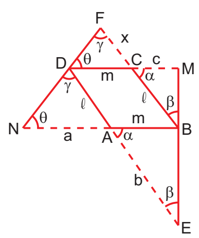
$9k = 4k + FE \Rightarrow FE = 5k$

En el $\triangle BEM$, por el Teorema de Thales:

$$\frac{ED}{BD} = \frac{FE}{MF} \Rightarrow \frac{x}{8} = \frac{5k}{4k} \Rightarrow x = 10$$

Clave B

5.



De la gráfica:

$\triangle DFC \sim \triangle NDA$

$$\Rightarrow \frac{x}{m} = \frac{\ell}{a} \Rightarrow xa = \ell m \qquad \text{... (I)}$$

$\triangle CMB \sim \triangle ABE$

$$\Rightarrow \frac{c}{\ell} = \frac{m}{b} \Rightarrow cb = \ell m \qquad \text{... (II)}$$

Nos piden:

$$\frac{(AE)(MC)}{(CF)(AN)} = \frac{bc}{xa} \qquad \text{... (III)}$$

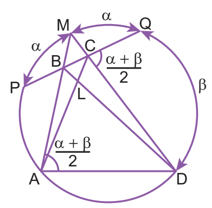
Reemplazando (I); (II) en (III):

$$\frac{bc}{xa} = \frac{\ell m}{\ell m} = 1$$

$$\therefore \frac{(AE)(MC)}{(CF)(AN)} = 1$$

Clave B

6.



Si $\widehat{PM} = \widehat{MQ} = \alpha$ y $\widehat{QD} = \beta$

Por ángulo inscrito y ángulo interno.

$$m\angle MAD = m\angle QCD = \frac{\alpha+\beta}{2}$$

Como las $m\angle MAD$ y $m\angle QCD$ son iguales, entonces el ⏢ABCD es inscriptible.

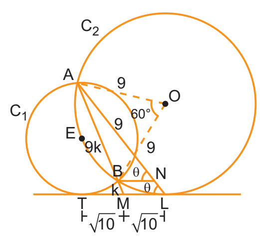
Por el teorema de las cuerdas en el ⏢ABCD:

$(BL)(LD) = (AL)(LC)$ y por dato $(AL)(LC) = 1$

$\Rightarrow (BL)(LD) = 1$

Clave C

7.



Trazamos $\overline{AM}$ que contiene el punto B.

Por el teorema de la tangente

De C_1: $(TM)^2 = (AM)(BM)$ ... (α)

De C_2: $(ML)^2 = (AM)(BM)$ ... (β)

$\Rightarrow$ de (α) y (β) se deduce:

$$TM = ML = \frac{2\sqrt{10}}{2} = \sqrt{12}$$

Por el Teorema de Thales en el $\triangle MAL$, dado que $\overline{BN} \parallel \overline{TL}$

$$\frac{NL}{AN} = \frac{BM}{AB} = \frac{1}{9}$$

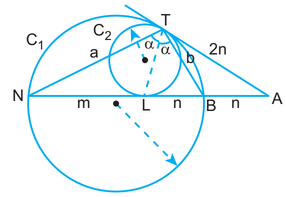
En (α): $(\sqrt{10})^2 = k(10k) \Rightarrow k = 1$

Si $x = 1$, el $\triangle AOB$ es equilátero:

$\Rightarrow m\angle AOB = 60°$

Clave B

8.



Por el Teorema de la tangente:

C_2: $TA = LA = 2n$ ... (I)

C_1: $(TA)^2 = (AB)(AN)$ ... (II)

De (I) y (II):

$(2n)^2 = n(2n + m)$

$4n^2 = n(2n + m)$

$\Rightarrow 2n = m$ ...(III)

Por propiedad; $m\angle NTL = m\angle LTB = \alpha$

En el $\triangle NTB$ por el teorema de la bisectriz interior:

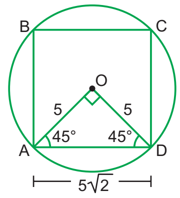
$\frac{a}{b} = \frac{m}{n}$, reemplazando en (III):

$$\frac{a}{b} = \frac{2n}{n} = 2$$

Clave A

APLICAMOS LO APRENDIDO (página 77) Unidad 4

1.



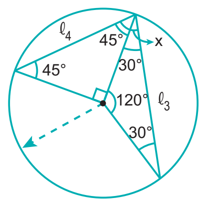
Perímetro del cuadrado:

$2p_{ABCD} = 4(5\sqrt{2})$

$2p_{ABCD} = 20\sqrt{2}$ m

Clave E

2.

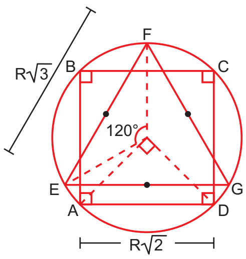


$x = 45° + 30°$

$x = 75°$

Clave C

3.

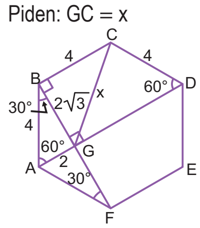


$2p_{EFG} = 3R\sqrt{3}$ y $2p_{ABCD} = 4R\sqrt{2}$

$\Rightarrow \dfrac{2p_{EFG}}{2p_{ABCD}} = \dfrac{3R\sqrt{3}}{4R\sqrt{2}} = \dfrac{3\sqrt{6}}{8}$

Clave C

4. Piden: GC = x



Por el teorema de Pitágoras en el ⊿GBC:

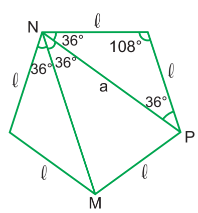
$x^2 = (2\sqrt{3})^2 + 4^2$

$x^2 = 12 + 16 = 28$

$\therefore x = 2\sqrt{7}$ cm

Clave A

5. Piden: ℓ



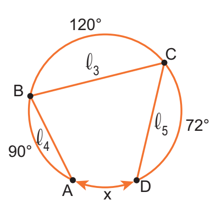
En el ΔMNP: Decágono regular.

$\alpha_{10} = 36°$, $R = a$

$\Rightarrow$ Por propiedad, sabemos: $\ell = a\left(\dfrac{\sqrt{5}-1}{2}\right)$

Clave C

6.



Como:

$BC = \ell_3 \Rightarrow m\overset{\frown}{BC} = 120°$

$AB = \ell_4 \Rightarrow m\overset{\frown}{AB} = 90°$

$CD = \ell_5 \Rightarrow m\overset{\frown}{CD} = 72°$

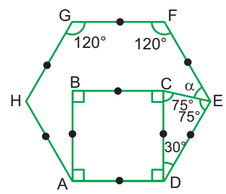
Luego:

$90° + 120° + 72° + x = 360°$

$282° + x = 360° \quad \therefore x = 78°$

Clave B

7.



Como el polígono ABCD es un cuadrado:

$\Rightarrow m\angle i = 90°$

Además, el polígono ADEFGH es un hexágono regular:

$\Rightarrow m\angle i = 120°$

Como el ΔDEC es isósceles

$\Rightarrow m\angle CED = m\angle DCE = 75°$

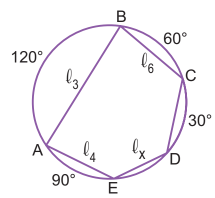
De la figura:

$m\angle FED = 120°$

$\alpha + 75° = 120° \qquad \therefore \alpha = 45°$

Clave C

8.



Como:

$AB = \ell_3 \Rightarrow m\overset{\frown}{AB} = 120°$

$AE = \ell_4 \Rightarrow m\overset{\frown}{AE} = 90°$

$BC = \ell_6 \Rightarrow m\overset{\frown}{BC} = 60°$

Luego:

$90° + 120° + 60° + 30° + m\overset{\frown}{ED} = 360°$

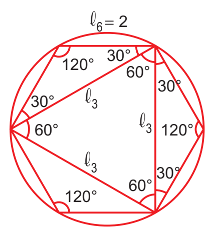
$300° + m\overset{\frown}{ED} = 360°$

$\Rightarrow m\overset{\frown}{ED} = 60°$

$\dfrac{360°}{x} = 60° \qquad \therefore x = 6$

Clave A

9.



$6\ell_6 = 12$

$\Rightarrow \ell_6 = 2$

$\therefore R = 2$

Luego:

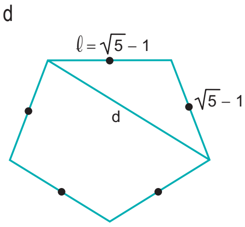
$\ell_3 = R\sqrt{3}$

$\ell_3 = 2\sqrt{3}$

$\therefore 3\ell_3 = 6\sqrt{3}$

Clave B

10. Piden: d

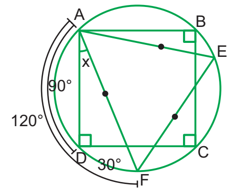


Por la propiedad del decágono regular:

$\ell = d\left(\dfrac{\sqrt{5}-1}{2}\right) = \sqrt{5} - 1 \qquad \therefore d = 2$

Clave C

11.



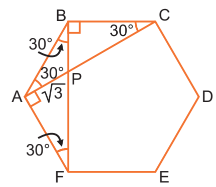
Por ángulo inscrito:

$2x = 30°$

$x = 15°$

Clave B

12.



En el ⊿PAF: $AF = (\sqrt{3})(\sqrt{3}) = 3$

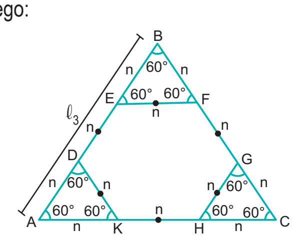
$\therefore 2p_{ABCDEF} = 6(3) = 18$

Clave D

13. El radio de la circunferencia mide 6, entonces el lado del triángulo equilátero será:

$\ell_3 = R\sqrt{3} = 6\sqrt{3}$

Luego:



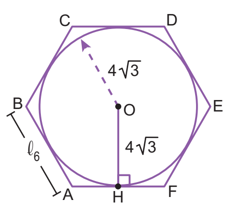
Por dato: el hexágono DEFGHK es regular.

Del gráfico: $\ell_3 = 3n$

$\Rightarrow 6\sqrt{3} = 3n \qquad \therefore n = 2\sqrt{3}$

Clave E

14.



Por dato: ABCDEF es un hexágono regular

Del gráfico:

OH: apotema del hexágono regular

$\Rightarrow OH = ap_6 = 4\sqrt{3}$

Sabemos:

$\ell_6 = R \quad ...(1) \quad \wedge \quad ap_6 = \dfrac{R\sqrt{3}}{2} \quad ...(2)$

Dividiendo (1) y (2):

$\dfrac{\ell_6}{ap_6} = \dfrac{2}{\sqrt{3}} \Rightarrow \ell_6 = \dfrac{2\sqrt{3}\,ap_6}{3} = \dfrac{2\sqrt{3}(4\sqrt{3})}{3}$

$\therefore \ell_6 = 8$

Clave D

PRACTIQUEMOS

Nivel 1 (página 79) Unidad 4

Comunicación matemática

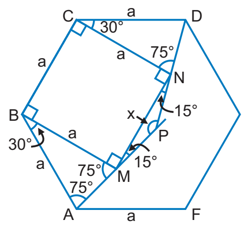
1.

2.

3.

Razonamiento y demostración

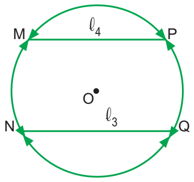
4.



En el ΔMNP:
$x + 15° + 15° = 180°$
$\Rightarrow x = 150°$

Clave D

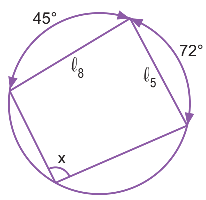
5.



Por dato:
$MP = \ell_4 \Rightarrow \overset{\frown}{MP} = 90°$
$NQ = \ell_3 \Rightarrow \overset{\frown}{NQ} = 120°$
$\overline{MP} \parallel \overline{NQ} \Rightarrow m\overset{\frown}{MN} = m\overset{\frown}{PQ} = x$
Luego: $2x + 90° + 120° = 360°$
$\therefore x = 75°$

Clave D

6.

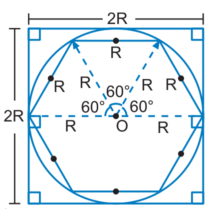


Por ángulo inscrito:
$\therefore x = \frac{1}{2}(45° + 72°)$
$x = 58{,}5°$

Clave D

Resolución de problemas

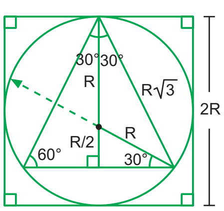
7.



Del gráfico:
Lado del cuadrado = 2R
Lado del hexágono = R
$\therefore \dfrac{\text{lado del cuadrado}}{\text{lado del hexágono}} = 2$

Clave B

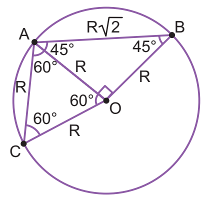
8.



Por dato:
$R + \frac{R}{2} = 12$
$R = 8$
$\therefore 2R = 16$

Clave D

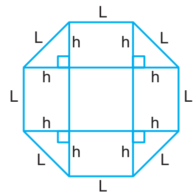
9.



Del gráfico:
$AB = \ell_4$
$AC = \ell_6$
$\therefore m\angle BAC = 105°$

Clave B

10. Del gráfico:

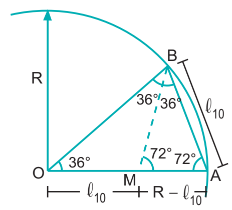


$L = h\sqrt{2}$

$\Rightarrow h = \dfrac{L\sqrt{2}}{2}$

Clave E

11.



Trazamos la bisectriz BM del $\angle$OBA.
$\Rightarrow AB = MB = OM = \ell_{10}$
Por propiedad de semejanza:
$(\ell_{10})^2 = R(R - \ell_{10})$
Resolviendo:
$\therefore \ell_{10} = \frac{R}{2}(\sqrt{5} - 1)$

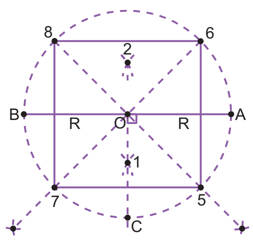
Clave D

Nivel 2 (página 79) Unidad 4

Comunicación matemática

12.

13.



Primero, con radio mayor que R y con centros en A y B determinamos los puntos 1 y 2; luego al prolongar la recta $\overleftrightarrow{12}$ obtenemos el punto C sobre la circunferencia; luego manteniendo el mismo radio y con centros en B y C obtenemos el punto 3; de la misma manera obtenemos el punto 4, finalmente las flechas $\overleftrightarrow{3O}$ y $\overleftrightarrow{4O}$ determinan sobre la circunferencia los puntos 5; 6 y 7; 8 respectivamente, los cuales son los vértices del cuadrado.

Razonamiento y demostración

14. Se nota que:

$m\overset{\frown}{AB} = 60°$; $m\overset{\frown}{DC} = 120°$; $m\overset{\frown}{AD} = m\overset{\frown}{BC}$

$\Rightarrow m\overset{\frown}{AD} + m\overset{\frown}{BC} + 60° + 120° = 360°$
$\downarrow$
$m\overset{\frown}{AD}$

Luego: $m\overset{\frown}{AD} = 90°$

Entonces: $x = \dfrac{\overset{\frown}{BC}}{2} = \dfrac{90°}{2} = 45°$

Clave A

15. De manera similar:

$x = \dfrac{360° - 90° - 120°}{2} = \dfrac{150°}{2}$

$\therefore x = 75°$

Clave B

16. De los datos:

$AB = \frac{R}{2}\sqrt{10 - 2\sqrt{5}} = \ell_5 \Rightarrow m\overset{\frown}{AB} = 72°$

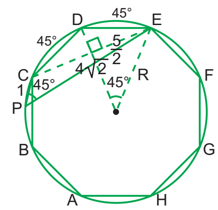
$BC = \frac{R}{2}(\sqrt{5} - 1) = \ell_{10} \Rightarrow m\overset{\frown}{BC} = 36°$

Luego: $2m\angle ABC + m\overset{\frown}{AB} + m\overset{\frown}{BC} = 360°$
$2m\angle ABC + 72° + 36° = 360°$
$\therefore m\angle ABC = 126°$

Clave A

Resolución de problemas

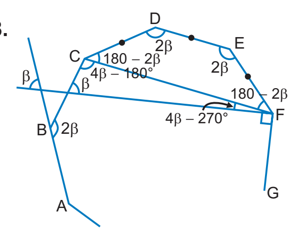
17.



En el ΔPCE:
$(CE)^2 = 1 + (4\sqrt{2})^2 - 8\sqrt{2}\cos 45°$
$(CE)^2 = 33 - 8$
$CE = 5$
$\therefore R = \frac{5}{2}\sqrt{2}$

Clave E

18.

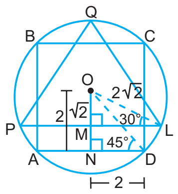


$\overline{CF} \parallel \overline{DE}$
$\Rightarrow m\angle DCF = m\angle EFC = 180° - 2\beta$
$\beta + 4\beta - 180° + 4\beta - 270° = 180°$
$9\beta = 630°$
$\beta = 70°$
$2\beta = 140°$

Luego: $\dfrac{180°(n - 2)}{n} = 140°$
$9n - 18 = 7n$
$2n = 18$
$n = 9$

Clave E

19.



$OM = ap_{\triangle PQL}$

$\sqrt{2} = R/2$

$\Rightarrow R = 2\sqrt{2}$

En el ⊿OND:

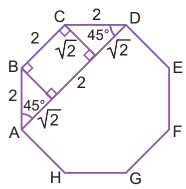
$ON = \dfrac{2\sqrt{2}}{\sqrt{2}} = 2$

$MN = ON - OM$

$MN = 2 - \sqrt{2}$

Clave B

20.



Por dato:

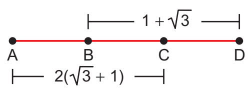
$8(AB) = 16$

$AB = 2$

$\therefore AB = 2\sqrt{2} + 2$

Clave C

21.



$\overline{AB}$ es la sección áurea de $\overline{AC}$, entonces:

$AB = \left(\dfrac{\sqrt{5} - 1}{2}\right)(2)(\sqrt{3} + 1)$

$AB = \sqrt{15} + \sqrt{5} - \sqrt{3} - 1$ (+)

$BD = \sqrt{3} + 1$

$\therefore AD = \sqrt{15} + \sqrt{5}$

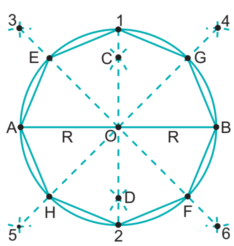
Clave B

Nivel 3 (página 80) Unidad 4

Comunicación matemática

22.

23.

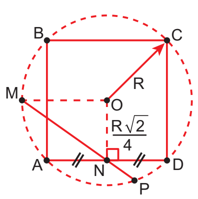


Primero con un radio mayor que R y con centros en A y B determinamos los puntos C y D; luego al prolongar $\overline{OC}$ y $\overline{OD}$ obtenemos sobre la circunferencia los puntos 1 y 2 respectivamente.

Luego con radio R y con centros en A y 1 determinamos el punto externo 3, repetimos esta operación con centros en 1 y B, luego con centros en B y 2, y finalmente centros en 2 y A para determinar los puntos externos 4; 6 y 5; unimos 3 con 6 y 5 con 4 para hallar sobre la circunferencia los puntos E; F y G; H, los cuales junto con los puntos A; B; 1 y 2 son los vértices de un octógono regular.

Razonamiento y demostración

24.



Del gráfico:

$AD = \ell_4 = R\sqrt{2}$

$\Rightarrow AN = ND = \dfrac{R}{2}\sqrt{2}$

Además:

$ON = ap_4 = \dfrac{R\sqrt{2}}{4}$

En el ⊿MON: (T. Pitágoras):

$(ON)^2 + (OM)^2 = (MN)^2$

$\left(\dfrac{R\sqrt{2}}{4}\right)^2 + R^2 = (MN)^2$

$\therefore MN = \dfrac{R\sqrt{6}}{2}$

Por el teorema de las cuerdas:

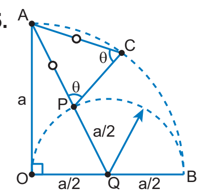
$(NP)(MN) = (AN)(ND)$

$(NP)\left(\dfrac{R\sqrt{6}}{2}\right) = \left(\dfrac{R\sqrt{2}}{2}\right)\left(\dfrac{R\sqrt{2}}{2}\right)$

$\therefore NP = \dfrac{R\sqrt{6}}{6}$

Clave C

25.



Sea $OB = a$

$\Rightarrow OQ = QB = a/2$

$\Rightarrow$ ⊿AOQ es notable de $53°/2$ y $127°/2$

$\therefore AQ = a$

En el $\triangle APC$ (isósceles):

$AC = AQ - PQ$

$AC = \dfrac{a\sqrt{5}}{2} - \dfrac{a}{2} = \dfrac{a}{2}(\sqrt{5} - 1)$

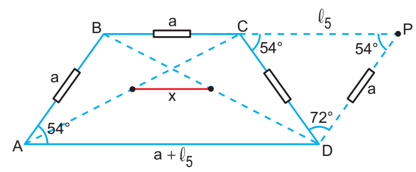
$\Rightarrow AC = \ell_{10}$ $\quad \therefore m\overset{\frown}{AC} = 36°$

$\therefore m\overset{\frown}{AC} + m\overset{\frown}{CB} = 90°$

$\Rightarrow m\overset{\frown}{CB} = 90° - 36° \Rightarrow m\overset{\frown}{CB} = 54°$

Clave B

26. Prolongamos $\overline{BC}$ hasta P tal que: $\overline{PD} // \overline{AB}$



Luego $m\angle PCD = m\angle CPD = 54°$

$\Rightarrow CP = \ell_5 = \dfrac{a}{2}\sqrt{10 - 2\sqrt{5}}$ ... (1)

Se sabe: $x = \dfrac{AD - BC}{2} \Rightarrow x = \dfrac{a + \ell_5 - a}{2}$

$\therefore x = \dfrac{\ell_5}{2}$

De (1): $x = \dfrac{a}{4}\sqrt{10 - 2\sqrt{5}}$; del dato:

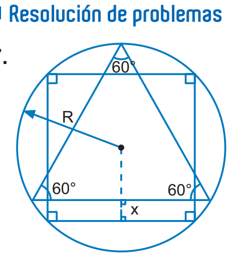
$a = \sqrt{10 + 2\sqrt{5}}$

$\Rightarrow x = \dfrac{(\sqrt{10 + 2\sqrt{5}})(\sqrt{10 - 2\sqrt{5}})}{4} \therefore x = \sqrt{5}$

Clave D

Resolución de problemas

27.



$x = ap_4 - ap_3$

$x = \dfrac{R}{2}\sqrt{2} - \dfrac{R}{2}$

$= \dfrac{R}{2}(\sqrt{2} - 1)$ ...(1)

Dato:

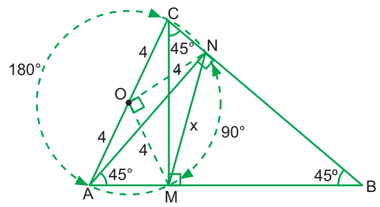
$R = \sqrt{2} + 1$

Reemplazando en (1):

$\therefore x = \dfrac{(\sqrt{2}+1)(\sqrt{2}-1)}{2} = 0,5$

Clave A

28.



Por dato: $m\angle ABC = 45°$

$\Rightarrow m\angle NAB = m\angle MCB = 45°$

$\Rightarrow$ ⏢ACNM es inscriptible

Entonces:

$m\angle CNA = m\angle CMA = 90°$

$\Rightarrow \overline{AC}$ es diámetro (O: centro)

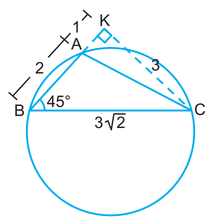
$ON = OM = 4$

Por ángulo central: $m\angle NOM = 90°$

En el ◺NOM isósceles: $x = 4\sqrt{2}$

Clave D

29.



$m\overset{\frown}{AC} = 90° \Rightarrow AC = \ell_4 = R\sqrt{2}$

En el ◺AKC por Pitágoras: $AC = \sqrt{10}$

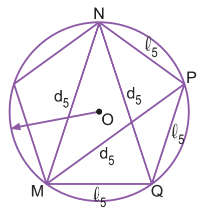
Reemplazando:

$R\sqrt{2} = \sqrt{10}$

$\therefore R = \sqrt{5}$ m

Clave C

30.



En el trapecio isósceles MNPQ, aplicamos el teorema de Ptolomeo:

$(d_5)^2 = d_5(\ell_5) + (\ell_5)^2$

Resolviendo: $d_5 = \ell_5\left(\dfrac{\sqrt{5}+1}{2}\right)$

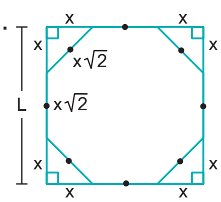
En el problema: $\ell_5 = \sqrt{5} - 1$

$\Rightarrow d_5 = \dfrac{(\sqrt{5}-1)(\sqrt{5}+1)}{2} = 2$

$\therefore \Sigma$ diagonales $= 5(d_5) = 5(2) = 10$ m

Clave B

31.



Del gráfico:

$x + x\sqrt{2} + x = L$

$\therefore x = \dfrac{L}{2}(2 - \sqrt{2})$

Clave A

APLICAMOS LO APRENDIDO (página 82) Unidad 4

1. Usando la fórmula trigonométca.

$$A_{\Delta} = \frac{(x)(4)}{2}\text{sen}45°$$

$$7\sqrt{2} = \frac{4x}{2} \cdot \frac{\sqrt{2}}{2}$$

$$7\sqrt{2} = x\sqrt{2}$$

$$\therefore\ x = 7 \text{ cm}$$

Clave D

2. En el ⊿ABC usamos el teorema de Pitágoras (r es el circunradio).

$$(7)^2 = (3 + r)^2 + (4 + r)^2$$

$$49 = 9 + 6 + r^2 + 16 + 8r + r^2$$

$$12 = (7 + r)r \qquad \text{... (1)}$$

Luego:

$$A_{⊿ABC} = pr\ ;\ p = \frac{1}{2}(7 + 4 + r + 3 + r)$$

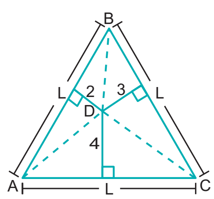
$$A_{⊿ABC} = (7 + r)\ ;\ \ p = 7 + r$$

Reemplazando en (1):

$$A_{⊿ABC} = 12\text{cm}^3$$

Clave A

3.



Del gráfico:

$$A_{\Delta ABC} = A_{\Delta ADB} + A_{\Delta BDC} + A_{\Delta ADC}$$

Entonces:

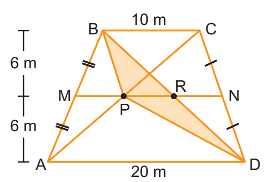
$$\frac{L^2\sqrt{3}}{4} = \frac{L(2)}{2} + \frac{L(3)}{2} + \frac{L(4)}{2}$$

$$\frac{L^2\sqrt{3}}{4} = \frac{9L}{2} \Rightarrow L = 6\sqrt{3}$$

$$\therefore A_{\Delta ABC} = \frac{(6\sqrt{3})^2\sqrt{3}}{4} = 27\sqrt{3}\ \text{cm}^2$$

Clave A

4.



Por dato: ABCD es un trapecio.
Por propiedad:

$$PR = \frac{20 - 10}{2} \Rightarrow PR = 5$$

Del gráfico:

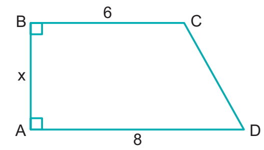
$$A_{\Delta PBD} = A_{\Delta PBR} + A_{\Delta PDR}$$

$$A_{\Delta PBD} = \frac{5(6)}{2} + \frac{5(6)}{2}$$

$$\therefore A_{\Delta PBD} = 30\ \text{m}^2$$

Clave B

5. Según el enunciado:



$$A = \left(\frac{6 + 8}{2}\right)x \ \Rightarrow\ 28 = \left(\frac{6 + 8}{2}\right)x$$

Resolviendo:

$$28 = 7x \Rightarrow x = 4 \text{ m}$$

Clave C

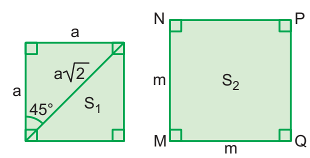
6. Área del cuadrilátero: $A = \frac{(d_1)(d_2)}{2}\text{sen}30°$

$$A = \frac{48}{2}\text{sen}30° = \frac{48}{2} \times \frac{1}{2} = 12$$

$$\therefore\ A = 12\ \text{m}^2$$

Clave D

7.



Por dato:

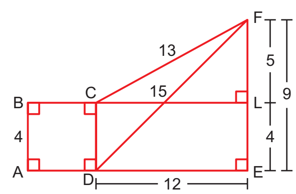
$$2S_1 = S_2 \Rightarrow 2(a^2) = m^2$$

$$a\sqrt{2} = m \ \Rightarrow\ 2p_{MNPQ} = 4m = 4a\sqrt{2}$$

$$\therefore p_{MNPQ} = 2a\sqrt{2}$$

Clave E

8.



Por teorema de Pitágoras:

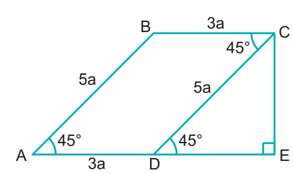
$$DE = 12$$

$$FL = 5 \ \Rightarrow\ LE = 4$$

$$\therefore\ S_{ABCD} = 4^2 = 16$$

Clave A

9.



$$2p_{ABCD} = 16a = 64$$

$$a = 4 \ \Rightarrow CE = \frac{5(4)}{\sqrt{2}} = 10\sqrt{2}$$

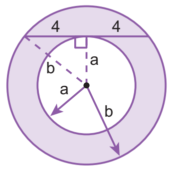
$$S_{ABCD} = 3a(10\sqrt{2})$$

$$S_{ABCD} = 3(4)(10\sqrt{2})$$

$$\therefore S_{ABCD} = 120\sqrt{2}\ \text{m}^2$$

Clave A

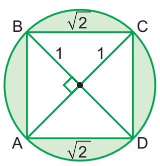
10. De la figura:



$A = \pi b^2 - \pi a^2 = \pi(b^2 - a^2)$
$\therefore\ A = \pi(4^2) = 16\pi\ m^2$

Clave D

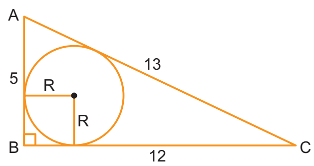
11. De la figura:



$A = \pi(1)^2 - (\sqrt{2})^2$
$\therefore\ A = \pi - 2$

Clave B

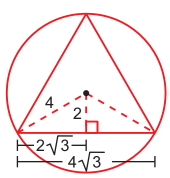
12. Según el enunciado:



Se cumple (por el teorema de Poncelet):
$AB + BC = AC + 2R$
$5 + 12 = 13 + 2R$
$R = 2$
$\therefore\ A = \pi R^2 = 4\pi\ cm^2$

Clave B

13. Sea A el área del círculo.
Del enunciado:



$A = \pi R^2 = \pi(4)^2 \Rightarrow A = 16\pi\ m^2$

Clave A

14. $A = S_{◺} - S_{◓}$

$$A = \frac{\pi(2)^2}{4} - \frac{\pi(1)^2}{2}$$

$$\therefore\ A = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

Clave A

PRACTIQUEMOS
Nivel 1 (página 84) Unidad 4

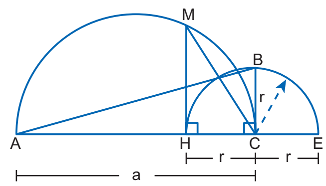
Comunicación matemática

1.

2.

Razonamiento y demostración

3.

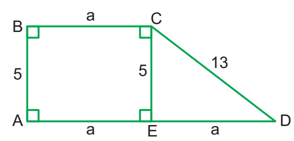


$A_{\triangle ABC} = \frac{ar}{2}$ ...(1)

En la semicircunferencia mayor; por propiedad:
$(MC)^2 = ar$
$(10)^2 = ar$
Reemplazando en (1):
$\therefore A_{\triangle ABC} = \frac{100}{2} = 50$

Clave D

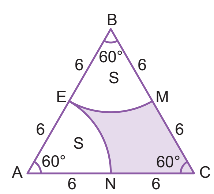
4.



Por el teorema de Pitágoras:
$13^2 = a^2 + 5^2 \Rightarrow a = 12$
Piden:
$A_{ABCD} = \frac{1}{2}(a + 2a)(BA)$
$\therefore A_{ABCD} = \frac{1}{2}(12 + 24)(5) = 90\ m^2$

Clave D

5.



Del gráfico:
$A_{somb.} = A_{\triangle ABC} - 2S$

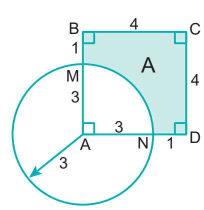
$$A_{somb.} = \frac{(12)^2\sqrt{3}}{4} - 2\left[\frac{\pi(6^2)60°}{360°}\right]$$

$A_{somb.} = 36\sqrt{3} - 12\pi$

$\therefore A_{somb.} = 12(3\sqrt{3} - \pi)$

Clave C

6.



Del gráfico:

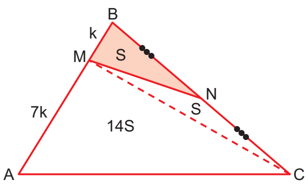
$A = (4)^2 - \dfrac{\pi(3)^2}{4}$

$A = 16 - \dfrac{9}{4}\pi$

Clave D

Resolución de problemas

7.



Para la mediana MN:

$S_{\triangle MBN} = S_{\triangle MNC} = S$

Para la ceviana CM:

$\dfrac{S_{\triangle CBM}}{S_{\triangle CMA}} = \dfrac{k}{7k} \Rightarrow \dfrac{2S}{S_{\triangle CMA}} = \dfrac{1}{7} \Rightarrow S_{\triangle CMA} = 14S$

Por dato: $S_{\triangle ABC} = 144$

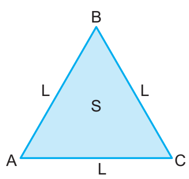
$\Rightarrow 16S = 144 \Rightarrow S = 9$

Piden: $S_{\triangle MBN} = S = 9$

$\therefore S_{\triangle MBN} = 9$

Clave C

8.



Sea el lado del triángulo equilátero: L

Por dato: $2p_{\triangle ABC} = S$

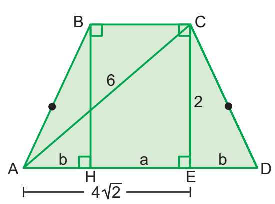
$3L = \dfrac{L^2\sqrt{3}}{4}$

$12 = L\sqrt{3}$

$\therefore\ L = 4\sqrt{3}$

Clave C

9.



Por dato: ABCD es un trapecio isósceles.

En el ◺AEC por el teorema de Pitágoras:

$AE = 4\sqrt{2}$

Piden:

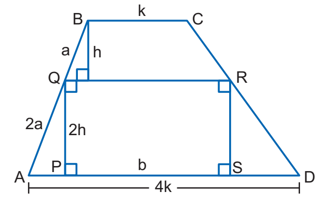
$A_{\square ABCD} = \dfrac{1}{2}(BC + AD)(CE)$

$A_{\square ABCD} = \dfrac{1}{2}(a + 2b + a)(2) = 2(a + b)$

$\therefore A_{\square ABCD} = 8\sqrt{2}\ \text{m}^2$

Clave A

10.



De la figura:

$A_{PQRS} = 2hb$

$A_{\square ABCD} = A_{\square AQRD} + A_{\square QBCR}$

$\dfrac{1}{2}(5k)(3h) = \dfrac{1}{2}(4k + b)(2h) + \dfrac{1}{2}(k + b)(h)$

$15k = 8k + 2b + k + b$

$\Rightarrow b = 2k$

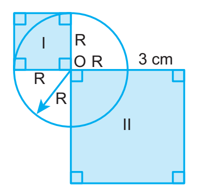
Piden:

$\dfrac{A_{\square PQRS}}{A_{\square ABCD}} = \dfrac{2hb}{\frac{1}{2}(5k)(3h)} = \dfrac{4b}{15k} = \dfrac{4(2k)}{15k}$

$\therefore \dfrac{A_{\square PQRS}}{A_{\square ABCD}} = \dfrac{8}{15}$

Clave E

11.



Por dato: $A_{\odot} = 9\pi$

$\Rightarrow \pi R^2 = 9\pi \Rightarrow R = 3$

Además: las regiones (I) y (II) son cuadradas.

$\Rightarrow A_{(I)} = R^2 = (3)^2 = 9$

$\Rightarrow A_{(II)} = (R + 3)^2 = (3 + 3)^2 = 36$

Piden:

$A_{(I)} + A_{(II)} = 9 + 36$

$\therefore A_{(I)} + A_{(II)} = 45\ \text{cm}^2$

Clave D

Nivel 2 (página 85) Unidad 4

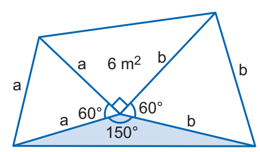
Comunicación matemática

12.

13.

Razonamiento y demostración

14. De la figura:



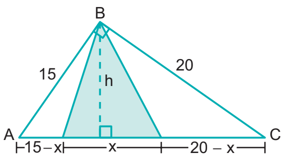
$\dfrac{ab}{2} = 6 \Rightarrow ab = 12$

$A_x = \dfrac{ab}{2}\text{sen}150° = \left(\dfrac{12}{2}\right)\left(\dfrac{1}{2}\right)$

$A_x = 3\ \text{m}^2$

Clave D

15. De la figura:



$15 - x + x + 20 - x = 25 \Rightarrow x = 10$

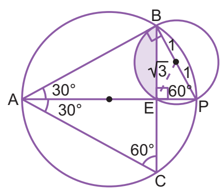
$h \,.\, (25) = 15\,(20) \Rightarrow h = 12$

Luego:

$A_{\triangle} = \dfrac{10 \times 12}{2} = 60 \text{ m}^2$

Clave A

16.



Del gráfico:

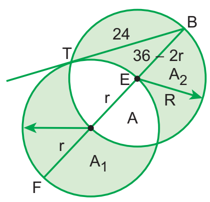
$A_{somb.} = A_{\frown BE}$

$A_{somb.} = \dfrac{\pi(1)^2 120°}{360°} - \dfrac{(1)^2}{2}\text{sen}120°$

$A_{somb.} = \dfrac{\pi}{3} - \dfrac{1}{2}\left(\dfrac{\sqrt{3}}{2}\right) = \dfrac{\pi}{3} - \dfrac{\sqrt{3}}{4}$

Clave B

17.



Por el teorema de la secante:

$(24)^2 = (36 - 2r)(36) \Rightarrow r = 10$

Además: $R = \dfrac{1}{2}(36 - r) = 13$

Del gráfico:

$A_2 + A = \pi R^2$ (–)

$A_1 + A = \pi r^2$

$A_2 - A_1 = \pi(R^2 - r^2)$

Entonces:

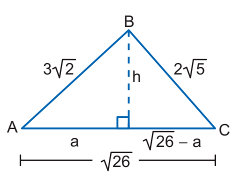
$A_2 - A_1 = \pi(13^2 - 10^2) = \pi(69)$

$\therefore A_2 - A_1 = 69\pi$

Clave E

Resolución de problemas

18.



$(3\sqrt{2})^2 - a^2 = (2\sqrt{5})^2 - (\sqrt{26} - a)^2$

$18 - a^2 = 20 - 26 - a^2 + 2\sqrt{26}\,a$

$24 = 2\sqrt{26}\,a$

$\dfrac{12}{\sqrt{26}} = a$

$h^2 = (3\sqrt{2})^2 - \left(\dfrac{12}{\sqrt{26}}\right)^2$

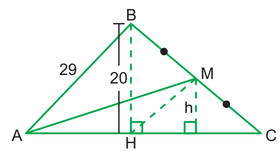
$h^2 = 18 - \dfrac{144}{26} = \dfrac{324}{26}$

$h = \dfrac{18}{\sqrt{26}}$

$\therefore S_{\triangle ABC} = \dfrac{\sqrt{26}(18)}{\sqrt{26}(2)} = 9$

Clave A

19.



$h = \dfrac{20}{2} = 10$

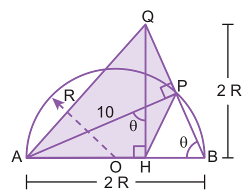
$AH^2 + 20^2 = 29^2$

$AH = 21$

$\therefore S_{\triangle AHM} = \dfrac{(21)(10)}{2} = 105$

Clave E

20.



Por propiedad:

$A_{\square AQPH} = \dfrac{(10)(2R)}{2}\text{sen}\theta$ ...(1)

En el ◺ APB: $\text{sen}\theta = \dfrac{10}{2R}$

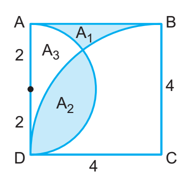
Reemplazando en (1):

$A_{\square AQPH} = \dfrac{(10)(2R)}{2} \cdot \left(\dfrac{10}{2R}\right) = \dfrac{100}{2} = 50$

$\therefore A_{\square AQPH} = 50 \text{ m}^2$

Clave D

21.



Del gráfico:

$A_2 + A_3 = \dfrac{\pi(2)^2}{2} = 2\pi$ ...(1)

$A_1 + A_3 = A_{\square} - A_{\triangleleft} = (4)^2 - \dfrac{\pi(4)^2}{4}$

$A_1 + A_3 = 16 - 4\pi \qquad ...(2)$

Restando (1) y (2):

$A_2 + A_3 - (A_1 + A_3) = 6\pi - 16$

$\therefore A_2 - A_1 = 2(3\pi - 8)$

Clave B

Nivel 3 (página 86) Unidad 4

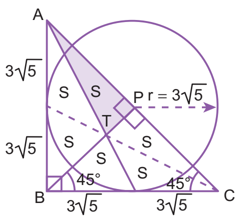
Comunicación matemática

22.

23.

Razonamiento y demostración

24.



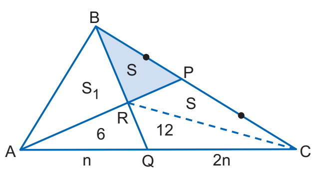
$$S_{\triangle ABC} = \frac{(6\sqrt{5})(6\sqrt{5})}{2} = 6S$$

$S = 15$

$\therefore S_{\triangle APT} = 15$

Clave D

25.



Para la ceviana RQ:

$S_{\triangle RQC} = 2S_{\triangle ARQ} \Rightarrow S_{\triangle RQC} = 12$

Para la mediana AP:

$S_1 + S = 18 + S \Rightarrow S_1 = 18$

Para la ceviana BQ:

$$\frac{S_1 + 6}{2S + 12} = \frac{n}{2n} \Rightarrow 2(18 + 6) = 2S + 12$$

$48 = 2S + 12$

$2S = 36$

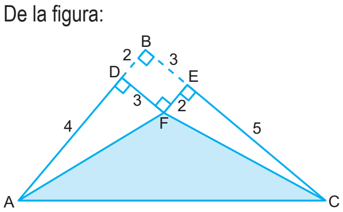
$S = 18$

Piden: $S_{\triangle RBP} = S$

$\therefore S_{\triangle RBP} = 18$

Clave D

26. De la figura:



El área pedida es:

$A_{\triangle ABC} - A_{\triangle ADF} - A_{\triangle EFC} - A_{\square DBEF}$

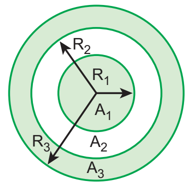
$$\frac{6 \times 8}{2} - \frac{4 \times 3}{2} - \frac{2 \times 5}{2} - 2 \times 3$$

Por lo tanto: $24 - 6 - 5 - 6 = 7$

El área pedida es: $7\ m^2$

Clave D

27.



Del gráfico:

$A_1 = \pi R_1^2;\ A_2 = \pi(R_2^2 - R_1^2);\ A_3 = \pi(R_3^2 - R_2^2)$

Por condición: $A_1 = A_2 = A_3$

Primero:

$A_1 = A_2 \Rightarrow \pi R_1^2 = \pi(R_2^2 - R_1^2)$

$R_1^2 = R_2^2 - R_1^2$

$2R_1^2 = R_2^2 \qquad ...(1)$

Luego:

$A_2 = A_3 \Rightarrow \pi(R_2^2 - R_1^2) = \pi(R_3^2 - R_2^2)$

$2R_2^2 - R_1^2 = R_3^2$

$2(2R_1^2) - R_1^2 = R_3^2$

$3R_1^2 = R_3^2 \qquad ...(2)$

De (1) y (2) se deduce:

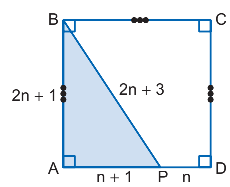
$$R_1^2 = \frac{R_2^2}{2} = \frac{R_3^2}{3} \Rightarrow \frac{R_1}{1} = \frac{R_2}{\sqrt{2}} = \frac{R_3}{\sqrt{3}}$$

$$\therefore R_1 = \frac{R_2}{\sqrt{2}} = \frac{R_3}{\sqrt{3}}$$

Clave C

Resolución de problemas

28.



Por dato: $AP - PD = 1 \wedge BP - AB = 2$

Sea: $PD = n \Rightarrow AP = n + 1$

Del gráfico: $AD = AB = 2n + 1$

$\Rightarrow BP - (2n + 1) = 2 \Rightarrow BP = 2n + 3$

En el $\triangle$BAP por el teorema de Pitágoras:

$(2n + 1)^2 + (n + 1)^2 = (2n + 3)^2$

Resolviendo: $n = 7$

$\Rightarrow AB = 2(7) + 1 = 15 \wedge AP = (7) + 1 = 8$

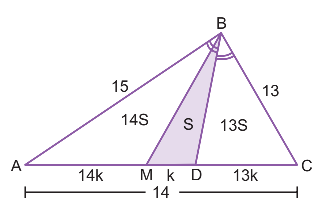
Piden:

$$S_{\triangle BAP} = \frac{(AB)(AP)}{2} = \frac{(15)(8)}{2}$$

$\therefore S_{\triangle BAP} = 60$

Clave C

29.



Por el teorema de la bisectriz interior:

$\dfrac{AD}{DC} = \dfrac{15}{13} = k \Rightarrow AD = 15k \wedge DC = 13k$

$\overline{BM}$ es mediana, entonces: AM = MC = 14k
Los triángulos ABM, MBD y DBC tienen la misma altura, entonces sus áreas son proporcionales a sus respectivas bases.

Para el $\triangle$ABC por la fórmula de Herón:

$p = \dfrac{13+14+15}{2} = 21$

$S_{\triangle ABC} = \sqrt{21(21-13)(21-14)(21-15)} = 84$

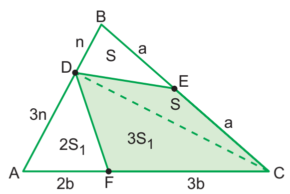
$28S = 84 \Rightarrow S = 3$

Piden: $S_{\triangle MBD} = S = 3$

$\therefore S_{\triangle MBD} = 3$

Clave A

30.



Por la ceviana CD:

$\dfrac{2S}{5S_1} = \dfrac{n}{3n} \Rightarrow S = \dfrac{5S_1}{6}$

Por dato: $A_{\triangle ABC} = 40\ cm^2$

$2S + 5S_1 = 40$

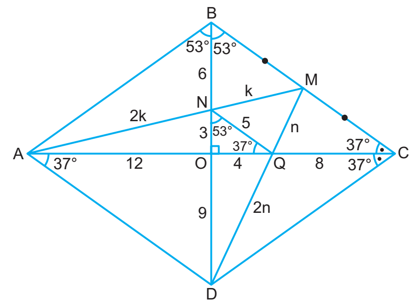
$2\left(\dfrac{5S_1}{6}\right) + 5S_1 = 40 \Rightarrow S_1 = 6 \wedge S = 5$

Piden: $A_{\square FDEC} = 3S_1 + S = 3(6) + 5$

$\therefore A_{\square FDEC} = 23\ cm^2$

Clave E

31.



Del gráfico:
O: punto de intersección de las diagonales
$\Rightarrow AO = OC \wedge BO = OD$
N: baricentro del $\triangle$ABC
Q: baricentro del $\triangle$BDC

En el $\triangle$AMD se cumple la relación de Tales.
$\Rightarrow \overline{AD} \parallel \overline{NQ}$

Luego en el $\triangle$ NOQ notable de 37° y 53°:
$OQ = 4 \Rightarrow QC = 8$
$ON = 3 \Rightarrow BN = 6$

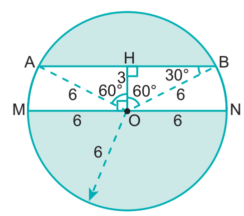
Piden:

$A_{\diamond ABCD} = \dfrac{(BD)(AC)}{2} = \dfrac{(18)(24)}{2}$

$\therefore A_{\diamond ABCD} = 216\ cm^2$

Clave B

32.



Del gráfico:
El $\triangle$OHB resulta ser notable de 30° y 60°.
Luego:

$A_{somb.} = A_{\frown AB} + A_{\frown MN}$

$A_{somb.} = \left[\dfrac{(120°)\pi(6)^2}{360°} - \dfrac{6^2 \text{sen} 120°}{2}\right] + \dfrac{\pi(6)^2}{2}$

$A_{somb.} = (12\pi - 9\sqrt{3}) + 18\pi$

$\therefore A_{somb.} = 30\pi - 9\sqrt{3}$

Clave D

RECTAS Y PLANOS EN EL ESPACIO

PRACTIQUEMOS

Nivel 1 (página 89) Unidad 4

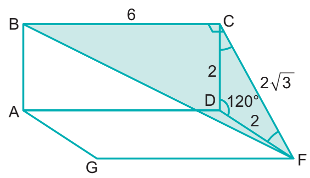
Comunicación matemática

1. AIV; BIII; CI; DV; EII

2. A) Plano
B) Triedro
C) Espacio
D) Diedro
E) Recta

Razonamiento y demostración

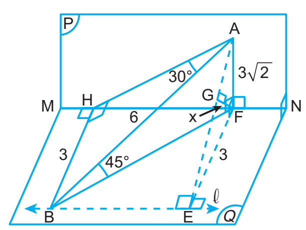
3.



En el $\triangle$CDF: CF = $2\sqrt{3}$ m
Luego en el ⊿BCF; por el Teorema de Pitágoras:
$BF^2 = 6^2 + (2\sqrt{3})^2$
$\therefore$ BF = $4\sqrt{3}$ m

Clave B

4.



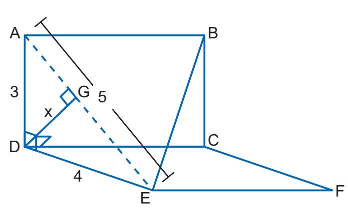
- Proyección de $\overline{AB}$ sobre el plano Q es BF.
- Se traza ℓ // $\overline{MN}$
- ⊿AFE $\perp$ plano Q.
- Como $\overline{AB} \in$ plano $\overline{ABE}$ $\Rightarrow$ FG $\perp$ AB
- Luego la mínima distancia entre $\overline{AB}$ y $\overline{MN}$ es $\overline{FG}$
- Por relaciones métricas en el ⊿AFE:

$$\frac{1}{FG^2} = \frac{1}{(3\sqrt{2})^2} + \frac{1}{(3)^2} \Rightarrow \frac{1}{FG^2} = \frac{1}{18} + \frac{1}{9}$$

$\therefore$ FG = $\sqrt{6}$ m

Clave C

5.

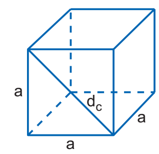


Datos: AD = 3 m y DE = 4 m, además el diedro $\overline{CD}$ mide 90°. Luego en el ⊿ADE: AE = 5m.
Se traza $\overline{DG}$ $\perp$ al plano ABE; pero como $\overline{BE}$ pertenece a dicho plano, entonces $\overline{DG} \perp \overline{BE}$.
Luego la mínima distancia entre $\overline{CD}$ y $\overline{BE}$ es DG.
En el ⊿ADE; por relaciones métricas:
(3)(4) = 5x.
$\therefore$ x = 2,4 m

Clave A

Resolución de problemas

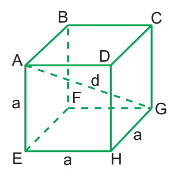
6.



Por dato el volumen del cubo es 27:
$\Rightarrow$ V = 27
$a^3 = 27 \Rightarrow a = 3$
Piden: la diagonal de una cara (d_c).
$\Rightarrow d_c = a\sqrt{2}$
$\therefore d_c = 3\sqrt{2}$

Clave C

7.



Un cubo presenta 4 diagonales de igual medida: $\overline{AG}$, $\overline{EC}$, $\overline{BH}$ y $\overline{FD}$.
Por dato:
$4d = 20\sqrt{3}$
$\Rightarrow d = 5\sqrt{3}$
Entonces:
$a\sqrt{3} = 5\sqrt{3}$
$\Rightarrow a = 5$
Piden:
$A_T = 6a^2 = 6(5)^2$
$\therefore A_T = 150$

Clave C

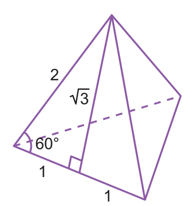
8. El tetraedro, el octaedro y el icosaedro.

Clave C

9. $A_T = a^2\sqrt{3} = 16\sqrt{3}\ u^2$

Clave A

10.



$$V = \frac{(2)^3\sqrt{2}}{12} = \frac{2\sqrt{2}}{3}\ m^3$$

Clave B

Nivel 2 (página 90) Unidad 4

Comunicación matemática

11.
I. (F) No necesariamente son colineales.
II. (V) Un hexaedro irregular.
III. (F) Tienen que ser colineales.
IV. (V) Si fuese perpendicular al plano.

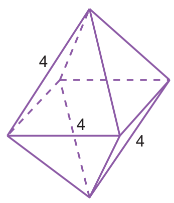
12.
I. F, son 5
II. V, cumple la definición de prisma
III. F, pertenecen a diferentes planos
IV. V, son colineales

Clave B

13.
I. (V), porque por dos puntos pasa una recta.
II. (V), porque dos planos están formados por más de 4 puntos.
III. (F), porque solo forman planos el resto del espacio queda vacío.
IV. (F), si fuesen paralelos definirían un plano.

Razonamiento y demostración

14.



Piden:

$A_T = 2(4)^2\sqrt{3} = 32\sqrt{3}$

$\therefore A_T = 32\sqrt{3}$

Clave C

15. $A = a^2\sqrt{3} = 49\sqrt{3} \Rightarrow a = 7$

$V = a^3 = 343\ m^3$

Clave A

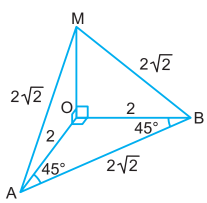
16. $d = a\sqrt{3}$, $a = 1 \Rightarrow d = \sqrt{3}$

$A_T = d^2\sqrt{3} = 3\sqrt{3}$

Clave B

Resolución de problemas

17.

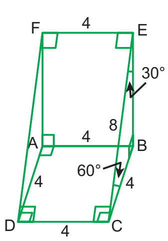


El $\triangle ABM$ es equilátero, entonces:

$$A_{\triangle ABM} = \frac{(2\sqrt{2})^2\sqrt{3}}{4} = 2\sqrt{3}$$

Clave C

18.



En el ⊿EBC de 30° y 60°:

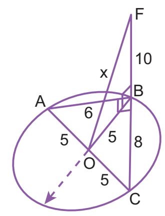
$EC = 2(4) = 8$

Por lo tanto:

$A_{\square FECD} = 4(8) = 32$

Clave A

19.



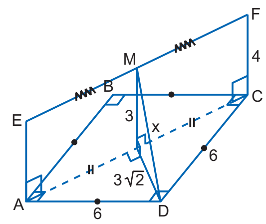
En el ⊿FBO:

$x = \sqrt{10^2 + 5^2}$

$x = 5\sqrt{5}$

Clave A

20.



Del gráfico:

$x^2 = 3^2 + (3\sqrt{2})^2$

$x = 3\sqrt{3}$

Clave C

Nivel 3 (página 90) Unidad 4

Comunicación matemática

21.
I. (F) Todos sus lados son congruentes.
II. (F) Porque, sus caras son polígonos.
III. (V) Porque, puede pertenecer a uno como a varios planos.
IV. (F) Porque, por un punto pasan infinitos planos.

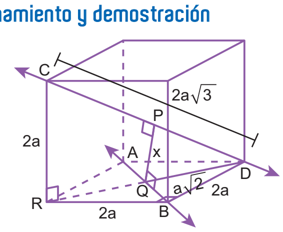
Clave C

22.
I. (V) por definición
II. (V) por definición
III. (V) siempre es un círculo
IV. (F) porque es un punto

Clave D

Razonamiento y demostración

23.



La menor distancia entre las rectas AB y CD es $\overline{PQ}$. Luego por semejanza entre los triángulos rectángulos CRD y QPD; se tiene:

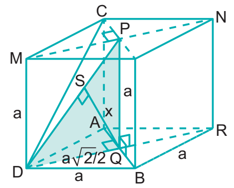
$$\frac{x}{2a} = \frac{a\sqrt{2}}{2a\sqrt{3}} \to x = \frac{a}{3}\sqrt{6} \quad \text{Pero:} a = \frac{\sqrt{6}}{2}\text{(Dato)}$$

Entonces: $x = \dfrac{\sqrt{6}}{2}\cdot\dfrac{\sqrt{6}}{3}$

$\therefore\ x = 1\ m$

Clave A

24.



Para determinar la mínima distancia entre $\overline{AB}$ y $\overline{CD}$ en el cubo, es necesario proyectar $\overline{CD}$ y $\overline{AB}$ sobre el plano DMNR, dichas proyecciones son $\overline{PD}$ y el punto Q respectivamente. Luego la menor distancia es el segmento perpendicular del punto Q a la proyección $\overline{PD}$, o sea $\overline{QS}$. Por relaciones métricas en el triángulo rectángulo DPQ:

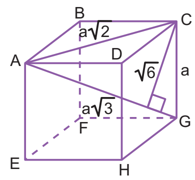
$$\frac{1}{x^2}=\frac{1}{a^2}+\frac{1}{\left(\frac{a\sqrt{2}}{2}\right)^2} \Rightarrow x=\left(\frac{a\sqrt{3}}{3}\right)$$

Pero: $a=\sqrt{3}$ (dato)

$\therefore\ x = 1$ m

Clave E

25.



$a(a\sqrt{2})=a\sqrt{3}(\sqrt{6})$

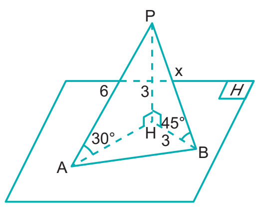
$a=3$

$\therefore V=a^3=27$

Clave A

Resolución de problemas

26.



Trazamos $\overline{PH}\perp$ plano H.

En el ⊿PHA notable de 30° y 60°:

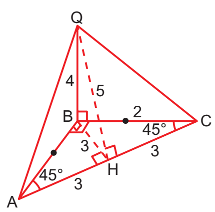
PH = 3

En el ⊿PHB notable de 45°:

$x=3\sqrt{2}$

Clave B

27.



Por propiedad:

AH = HC = BH = 3

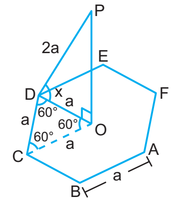
En el ⊿QBH:

QH = 5

$\therefore A_{\triangle AQC}=\frac{6(5)}{2}=15$

Clave B

28.



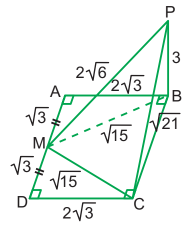
En el ⊿POD:

$DO=\frac{DP}{2}$

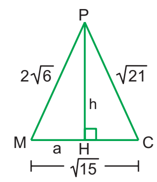
$\Rightarrow x = 60°$

Clave B

29.



En el $\triangle$PMC:



$$(\sqrt{21})^2=(2\sqrt{6})^2+(\sqrt{15})^2-2(\sqrt{15})(a)$$

$$21=24+15-2a\sqrt{15}$$

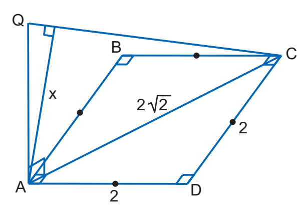
$$a=\frac{3\sqrt{15}}{5}$$

$$\Rightarrow h=\sqrt{(2\sqrt{6})^2-\left(\frac{3\sqrt{15}}{5}\right)^2}=\sqrt{\frac{93}{5}}$$

$$\therefore A_{\triangle PMC}=\sqrt{15}\cdot\sqrt{\frac{93}{5}}\cdot\frac{1}{2}=\frac{3\sqrt{31}}{2}$$

Clave D

30.



En el ⊿QAC por relaciones métricas:

$$\frac{1}{x^2}=\frac{1}{2^2}+\frac{1}{(2\sqrt{2})^2} \Rightarrow x=\frac{2\sqrt{6}}{3}$$

Clave C

APLICAMOS LO APRENDIDO (página 92) Unidad 4

1. Debe coincidir con la diagonal:

$$\sqrt{(2\sqrt{5})^2+(1)^2+(2)^2}$$
$$=\sqrt{20+1+4}=\sqrt{25}=5$$

Clave C

2. Es una pirámide inscrita:

$$V_p=\frac{V_{prisma}}{3}$$

$$V_p=\frac{192}{3}\ m^3=64\ m^3$$

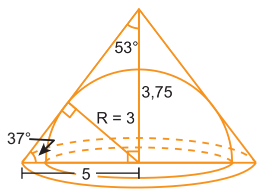
Clave C

3. Lado de la base (es un cuadrado)

$L=8$
$A_L=2ph=(8+8+8+8)4$
$=32(4)=128\ m^2$

Clave C

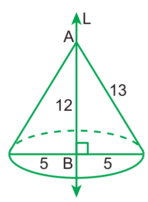
4. Según enunciado:



$$V_{hemisferio}=\frac{1}{2}\cdot\frac{4\pi}{3}R^3=\frac{2\pi}{3}(3)^3$$
$$\therefore\ V_{hemisferio}=18\pi$$

Clave C

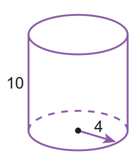
5.



$$V=\frac{1}{3}\pi(5^2)12=100\pi$$

Clave E

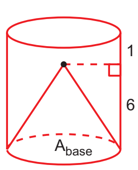
6.



a) $A_L=(2\pi)4\cdot 10=80\pi$
b) $A_T=80\pi+2\pi\cdot 4^2=112\pi$
c) $V=\pi\cdot 4^2\cdot 10=160\pi$

Clave C

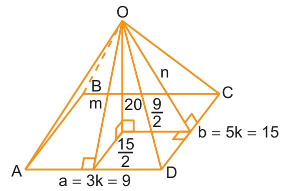
7. De la figura:



$$\frac{A_{base}\times 6}{3}=200\Rightarrow A_{base}=100$$
$$V_{cilindro}=Bh=100\times 7=700$$
$$V_{agua}=700-200=500$$

Clave C

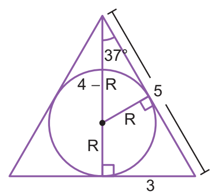
8.



Dato:
$2a+2b=48$
$a+b=24$
$3k+5k=24$
$k=3$
$\Rightarrow a=9\quad\wedge\quad b=15$
Aplicando Pitágoras:
$m=21{,}36\quad\wedge\quad n=20{,}5$
$$A_L=V=2\left(\frac{am}{2}+\frac{bn}{2}\right)=499{,}74$$

Clave A

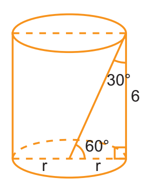
9.



$$\frac{R}{3}=\frac{4-R}{5}\quad\Rightarrow\quad R=\frac{3}{2}$$
$$A_{esfera}=4\pi R^2=4\pi\left(\frac{3}{2}\right)^2$$
$$\therefore\ A_{esfera}=9\pi\ m^2$$

Clave B

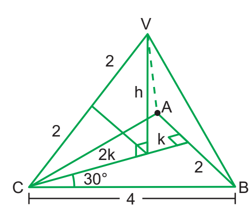
10.



$$6=k\sqrt{3}$$
$$\frac{6}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=k$$
$$\Rightarrow 2\sqrt{3}=k=r$$
$$\Rightarrow V=\pi(2\sqrt{3})^2 6$$
$$V=72\pi$$

Clave D

11.



$3k = 2\sqrt{3} \Rightarrow k = \dfrac{2\sqrt{3}}{3}$

Por Pitágoras:

$h^2 + (2k)^2 = (4)^2$

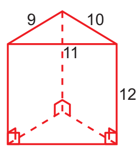
$h^2 + (4k)^2 = 16 \Rightarrow h = \dfrac{4\sqrt{2}}{\sqrt{3}}$

$$V = \frac{A_{base} \times h}{3} = \frac{\left(\frac{4^2\sqrt{3}}{4}\right) \times \frac{4\sqrt{2}}{\sqrt{3}}}{3}$$

$$V = \frac{16\sqrt{2}}{3}$$

Clave E

12.

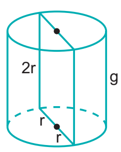


$A_L = (9 + 10 + 11)12 = 360\ m^2$

$V = \sqrt{15(6)(5)(4)}\,.12 = 360\sqrt{2}\ m^3$

Clave A

13.



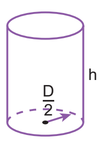
$\pi r^2(2r) = 16\pi$

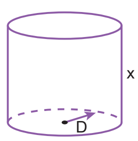
$r = 2$

$\therefore g = 2r = 2(2) = 4$

Clave C

14.





$V_1 = \pi\left(\dfrac{D}{2}\right)^2 h$ $\qquad V_2 = \pi D^2 x$

Por dato:

$V_1 = V_2$

$\pi\dfrac{D^2}{4}h = \pi D^2 x$

$x = \dfrac{h}{4}$

Clave C

PRACTIQUEMOS

Nivel 1 (página 94) Unidad 4

Comunicación matemática

1. Prisma: dos bases, lados poligonales, base poligonal.

Cilindro: dos bases, base circular, lado curva

Esfera: un semicírculo.

Cono: un vértice, una base, lado curvo, base circular

Pirámide: un vértice, una base, base circular, lado curvo.

2. V D; IV E; III A; IIC; IB

3. (V) Por definición

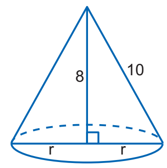
(V) Por definición

(F) Tiene base circular

(F) Tiene base circular

Razonamiento y demostración

4.



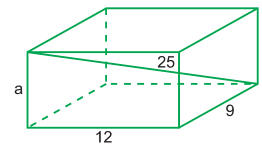
$r = 6$

$A_L = \pi rg$

$A_L = \pi(6)10 = 60\pi$

Clave A

5.



$25^2 = a^2 + 12^2 + 9^2$

$a = 20$

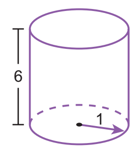
a) $A_L = 2(9\,.\,20 + 12\,.\,20) = 840$

b) $A_T = 2(9\,.\,20 + 12\,.\,20 + 9\,.\,12) = 1056$

c) $V = 9(12)(20) = 2160$

Clave E

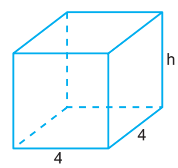
6.



$\therefore V = \pi(1^2)(6) = 6\pi\ m^3$

Clave E

7.



$4(4)(h) = 64$

$h = 4\ m$

Clave C

Resolución de problemas

8. Si la altura es h:

$V_{pirámide} = V_{cubo}$

$\dfrac{a^2h}{3} = a^3 \Rightarrow h = 3a$

Clave B

9. Si la base tiene n lados, entonces la pirámide tiene (2n) aristas.

$\Rightarrow 2n = 124 \Rightarrow n = 62$

El total de vértices es 63.

Clave B

10. Sean las dimensiones: a, b y c:

$ab = 6$

$bc = 10$

$ca = 15$

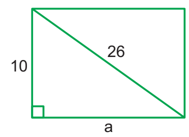
$a^2b^2c^2 = 900$

$\Rightarrow abc = 30$

$\therefore V = 3 \times 2 \times 5 = 30\ m^3$

Clave E

11.



$a^2 + 10^2 = 26^2$

$a = 24$

$\therefore A_L = 10(24) = 240\ cm^2$

Clave A

Nivel 2 (página 94) Unidad 4

Comunicación matemática

12.

I. (F) Su base región poligonal.
II. (V) Su base es región circular.
III. (F) Un poliedro tiene lados poligonales.
IV. (F) Una prisma tiene 2 bases y la piramide 1.

Clave C

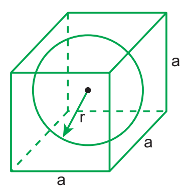
13. (NC) - (NC) - (C) - (NC)

14.

I. (F) base circular
II. (F) tiene que tener dos bases
III. (F) no necesariamente

Razonamiento y demostración

15.



$D = 6\sqrt{3} = a\sqrt{3}$

$a = 6$

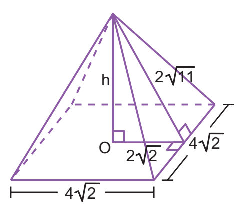
$\Rightarrow 2r = 6$

$r = 3$

$V_{esfera} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3^3) = 36\pi$

Clave B

16.



$h^2 + (2\sqrt{2})^2 = (2\sqrt{11})^2$

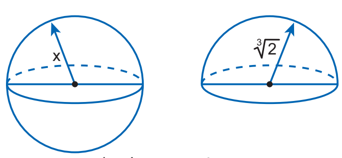
$h = 6$

$V = \frac{1}{3}(4\sqrt{2})^2(6)$

$V = 64$

Clave D

17.



$V_1 = \frac{4}{3}\pi x^3$ $\qquad V_2 = \frac{1}{2}\cdot\frac{4}{3}\pi(\sqrt[3]{2})^3$

$V_2 = \frac{4\pi}{3}$

$\Rightarrow \frac{4\pi}{3}x^3 = \frac{4\pi}{3}$

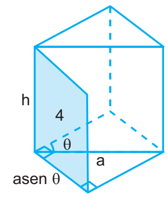
$x^3 = 1$

$x = 1$

Clave A

Resolución de problemas

18.



Por dato:

$A_{proyección} = 10$

$\Rightarrow a\text{sen}\theta \cdot h = 10$

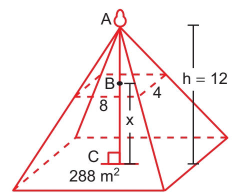
$V = A_{base} \cdot h$

$V = \frac{4 \cdot a\text{sen}\theta}{2} \cdot h = \underbrace{2a\text{sen}\theta \cdot h}_{10}$

$\therefore V = 20$

Clave D

19.



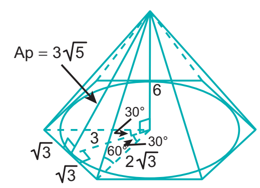
$\frac{8 \times 4}{288} = \frac{AB^2}{AC^2} = \frac{(12 - x)^2}{12^2}$

$16 = (12 - x)^2$

$4 = 12 - x \quad \Rightarrow x = 8$

Clave E

20.

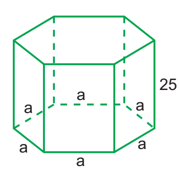


$A_L = 6\left(\frac{2\sqrt{3} \times 3\sqrt{5}}{2}\right)$

$A_L = 18\sqrt{15}$

Clave D

21.



$6a(25) = 1500$

$a = 10$

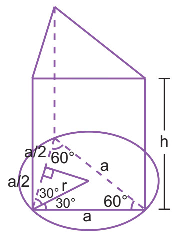
$\therefore A_T = 1500 + 2\left(6 \cdot \frac{10^2\sqrt{3}}{4}\right)$

$A_T = 1500 + 300\sqrt{3}$

$A_T = 300(\sqrt{3} + 5)\ m^2$

Clave A

22.



Por dato:

$a^3 = 9$

$h = 2(2r)$

$h = 4r$

$\Rightarrow h = \dfrac{4a}{\sqrt{3}}$

$V = a^2\dfrac{\sqrt{3}}{4}(h) = a^2\dfrac{\sqrt{3}}{4}\cdot\dfrac{(4a)}{\sqrt{3}} = a^3 = 9\ m^3$

Clave A

Nivel 3 (página 95) Unidad 4

Comunicación matemática

23.

I. (F) no necesariamente, generalmente es un triángulo isósceles.
II. (V) por definición.
III. (F) porque sería un cono.
IV. (F) por definición.

Clave C

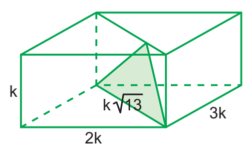
24.

I. (F) son romboides
II. (V) pueden ser polígonos regulares e irregulares.
III. (V)

Clave E

Razonamiento y demostración

25.



$k(2k)(3k) = 48$

$k = 2$

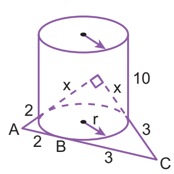
$A_{SOMBREADA} = \dfrac{(k\sqrt{13})k}{2}$

$= \dfrac{(2\sqrt{13})2}{2}$

$= 2\sqrt{13}\ m^2$

Clave D

26.



$(2 + x)^2 + (3 + x)^2 = 5^2$

$\Rightarrow x = 1$

Teorema de Poncelet:

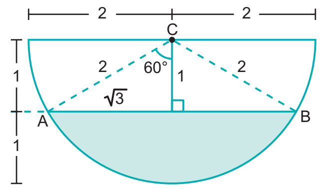
$3 + 4 = 5 + 2r$

$r = 1$

$\therefore V = \pi(1)^2 10 = 10\pi\ m^3$

Clave B

27. Una de las bases:



Área sombreada: $A_s = S_{\triangleleft ACB} - S_{\triangle ACB}$

$A_S = \dfrac{120°\pi 2^2}{360°} - \dfrac{2\sqrt{3}\times 1}{2}$

$A_S = \dfrac{4}{3}\pi - \sqrt{3}$

Volumen pedido: $V = A_{base}\times 6$

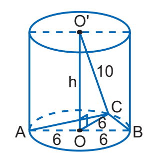
$V = \left(\dfrac{4}{3}\pi - \sqrt{3}\right)6$

$V = 8\pi - 6\sqrt{3}$

$V = 2(4\pi - 3\sqrt{3})$

Clave B

28.



$h^2 = 10^2 - 6^2$

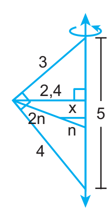
$h = 8$

$\therefore V = \pi(6)^2 . 8 = 288\pi$

Clave C

Resolución de problemas

29.



Calculamos la distancia del baricentro al eje de rotación.

$\dfrac{n}{x} = \dfrac{3n}{2,4}$

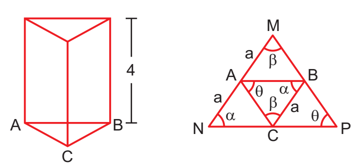
$x = \dfrac{4}{5}$

Teorema de Pappus:

$V_{GENERADO} = 2\pi\dfrac{(3\,.\,4)}{2}\cdot\dfrac{4}{5} = \dfrac{48\pi}{5}$

Clave C

30.

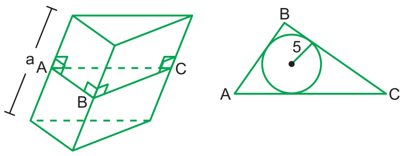


$A_{\Delta ABC} = \dfrac{A_{\Delta MNP}}{4} = \dfrac{36}{4} = 9$

$\therefore$ Volumen $= A_{\Delta ABC}(4) = 9(4) = 36\ m^3$

Clave B

31.



Sección recta: $\triangle ABC$

$A_{\triangle ABC} = p_{ABC}(5)$

$A_L = 2p_{S.R} \times a$

$48 = 2p_{S.R} \times a$

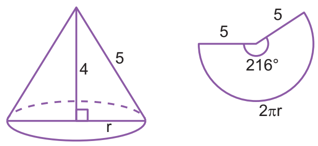
$24 = p_{S.R} \times a$

$a = \dfrac{24}{p_{ABC}}$

$\therefore V = A_{\triangle ABC} \times a = 5p_{ABC}\left(\dfrac{24}{p_{ABC}}\right) = 120\ m^3$

Clave E

32.



$5\left(\dfrac{6}{5}\pi\right) = 2\pi r \qquad 216° = \dfrac{6\pi}{5}$ rad

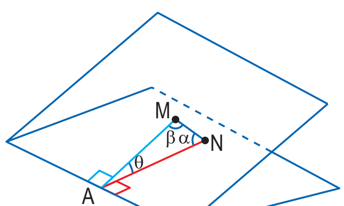
$r = 3$

$\therefore V = \dfrac{\pi(3^2)4}{3} = 12\pi\ m^3$

Clave A

MARATÓN MATEMÁTICA (página 97)

1.



Paso 1:

Por dato: $\dfrac{\theta}{2} = \dfrac{\alpha - \beta}{2} \Rightarrow \theta = \alpha - \beta$

$\therefore\ \alpha = \theta + \beta \Rightarrow \alpha = 90°$

Paso 2:

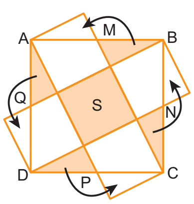
Si $\alpha = 90°$:

$\dfrac{\alpha}{\beta} = \dfrac{3}{2} \Rightarrow \beta = 30°$

$\therefore\ \theta = 90° - 30° = 60°$

Clave D

2. Por propiedad: el cuadrado es equivalente a la suma de los 5 cuadrados menores.



$\therefore\ 5S = 25^2$

$\Rightarrow S = 125\ m^2$

Clave C

3. Paso 1:

$V_{\text{esfera inicial}} = \dfrac{4}{3}\pi r^3;\ V_{\text{esfera final}} = \dfrac{4}{3}\pi(r + 0{,}01)^3$

Por dato:

$V_{\text{E final}} - V_{\text{E inicial}} = \dfrac{13}{3}\pi$

$\dfrac{4}{3}\pi(r + 0{,}01)^3 - \dfrac{4}{3}\pi r^3 = \dfrac{13}{3}\pi$

$\Rightarrow r = 1/2$

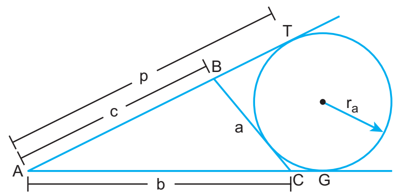
Paso 2:

$\text{Area}_{\text{inicial}} = \dfrac{4\pi}{4}\ ;\ V_{\text{E inicial}} = \dfrac{\pi}{6}$

$\text{Area}_{\text{inicial}} - V_{\text{E inicial}} = \pi - \dfrac{\pi}{6} = \dfrac{5\pi}{6}$

Clave A

4.



Paso 1:

Por propiedad:

$A_{\triangle ABC} = (p - a)r_a$

Donde:

P: semiperímetro $\triangle ABC$.

r_a: exradio relativo al lado a.

Paso 2:

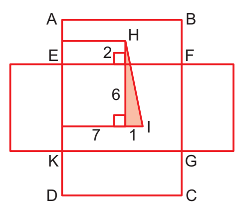
Datos del problema:

$p = 45$ m; $BC = a = 12$ m y $r_a = 10$ m

$\therefore\ A_{\triangle ABC} = (45 - 12)(10) = 330\ m^2$

Clave D

5.



1.er paso:

El área de la caja se desarrolla en un plano y ubicamos los puntos de recorrido de la hormiga ($\overline{HI}$).

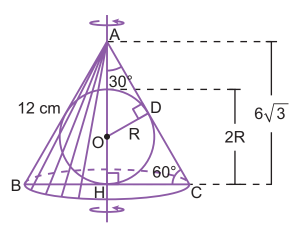
Paso 2:

Hallamos HI; por Pitágoras:

$HI^2 = 8^2 + 1^2 \Rightarrow HI = \sqrt{65}$ m

Clave D

6.



Paso 1: hallando los segmentos:

Del triángulo AOD. Por Pitágoras; AO = 2R.

$\Rightarrow AH = 3R$

Del triángulo AHB por Pitágoras; AH = $6\sqrt{3}$.

$\Rightarrow$ AH = 3R = $6\sqrt{3}$; R = $2\sqrt{3}$; BC = 12.

Paso 2: hallando los volúmenes:

$V_{cono} = \frac{1}{3}$ Área base $\times$ altura

$= \frac{1}{3}(\pi 6^2)(6\sqrt{3})$

$V_{cono} = 72\pi\sqrt{3}$

$V_{esfera} = \frac{4}{3}\pi R^3$

$= \frac{4}{3}\pi(2\sqrt{3})^3$

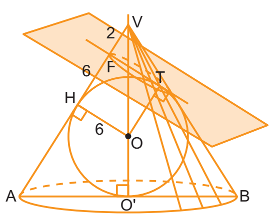
$V_{esfera} = 32\pi\sqrt{3}$

Paso 3:

$V_{cono} - V_{esfera} = 72\pi\sqrt{3} - 32\pi\sqrt{3} = 40\pi\sqrt{3}$ cm^3

Clave B

7.



Paso 1:

De la figura HFTO es un cuadrado.

$\Rightarrow$ el triángulo VHO es un triángulo notable de 37° en V.

$\therefore$ VO = 10 cm

Paso 2:

Halla la altura y el radio de la base del cono.

VO + OO' = 10 + 6 = 16 cm

$\Rightarrow$ por ángulo notable:

$\frac{AO'}{VO'} = \frac{3}{4} \Rightarrow \frac{AO'}{16} = \frac{3}{4}$

AO' = 12 cm = R de la base.

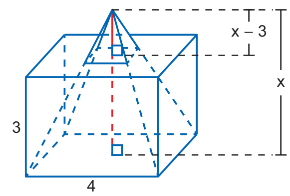
Paso 3:

$V_{cono} = \frac{1}{3}$ área base $\times$ altura

$= \frac{1}{3}(\pi\ 12^2)(16) = 768\pi$ cm^3

Clave A

8.



Paso 1: definimos términos;

Volumen común: V_{tronco} interior al paralelepípedo.

V_1: volumen de la pirámide mayor.

V_2: volumen de la pirámide menor.

x: altura de de la pirámide mayor.

x − 3: altura de la de pirámide menor.

$V_{tronco} = \frac{2}{3} V_{paralelepípedo}$

Paso 2°

$V_{paralelepípedo} = 4 \times 4 \times 3 = 48$

$V_{tronco} = \frac{2}{3}(4 \times 4 \times 3) = 32$

Por semejanza: $\frac{V_2}{V_1} = \frac{(x-3)^3}{x^3}$ ... (α)

Dato calculado: $V_1 - V_2 = V_{tronco} = 32$ ... (β)

$V_1 = \frac{1}{3}(4\ .\ 4)x$ ... (γ)

(β) ÷ (γ):

$\frac{V_1 - V_2}{V_1} = \frac{32}{\frac{16}{3}x} = \frac{6}{x} \Rightarrow 1 - \frac{V_2}{V_1} = \frac{6}{x}$ ... (θ)

(α) en (θ):

$1 - \frac{(x-3)^3}{x^3} = \frac{6}{x}$

$\frac{x-6}{x} = \frac{(x-3)^3}{x^3} \Rightarrow x - 6 = \frac{(x-3)^3}{x^2}$

$x^3 - 6x^2 = x^3 - 9x^2 + 27x - 27$

$\Rightarrow x^2 - 9x + 9 = 0$

$x = \frac{9 + \sqrt{45}}{2}$ cm

Clave E